



Price Analysis of Bitcoin: Volatility, Key Drivers and Evolution

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**Dissertation
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Biographic Note

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Pedro joined IBM International Services Center, in Slovakia, in 2015, and after a period of 5 months he was accepted for a 1 year internship at Siemens Healthcare, in Porto, where he worked in a Financial Analyst position.

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I also thank my family and closest friends for their most precious support and concern over my work.

Abstract

Bitcoin is currently the widest adopted of crypto-currencies, attracting diverse types of users and showing great volatility throughout its price history. This dissertation examines the Bitcoin price formation based on a set of drivers: from fundamentals in economic and financial literature to Bitcoin-specific variables. The obtained price formation model describes the relation of the several drivers with price behavior; while also establishing the price behavior near its long run equilibrium. Lastly, we also find evidence of asymmetrical impacts on price volatility caused by positive and negative shocks, supporting the previously described effect of the drivers.

Keywords: Bitcoin; Price Formation; Drivers; Volatility.

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Introduction

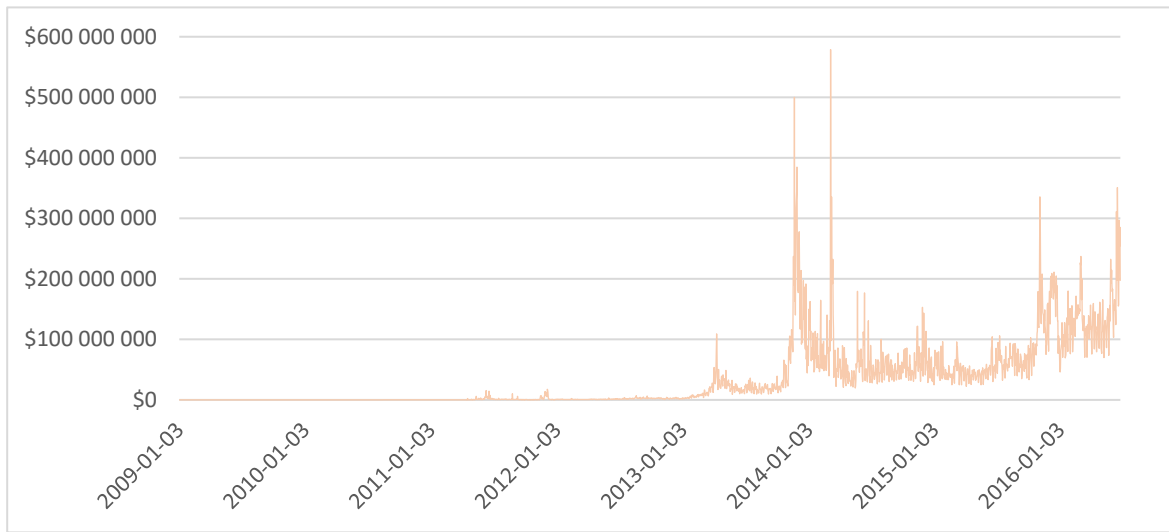
Crypto currencies, also generally called altcoins, have emerged as a new fascinating phenomenon in the financial markets. Amongst the many names of these type of assets currently in existence, such as the Auroracoin, Litecoin, Ethereum or even Dogecoin, Bitcoin stands as the most renowned in popularity, as well as higher market capitalization and trade volume.

Unlike a virtual coin or even electronic money – an Internet-based form of currency or medium of exchange, e.g. money balance recorded electronically on a debit card – a crypto currency makes use of cryptography to secure transactions and to control the creation of new units. They use a decentralized control, making it impossible to influence or control the supply of currency by printing units of fiat money. This long term contracting effective supply makes Bitcoin one of the only deflationary currency experiments in the world, which makes it similar to gold and other precious metals on that regard.

Bitcoin has experienced a remarkable growth in amount of users and general awareness since its inception in 2008, undoubtedly demonstrated by its clear upward trend in the number of daily transactions, transactions that reach daily values of around 150 million USD. This data can be seen in figure 1 shown below. Characteristics incorporated in the bitcoin technology, like its absolute transparency for every single transaction made, the low transaction costs or even the absence of fees and its controlled and known algorithm for currency creation may also constitute attraction points for users and investors.

Figure 1

Estimated Transaction Volume USD

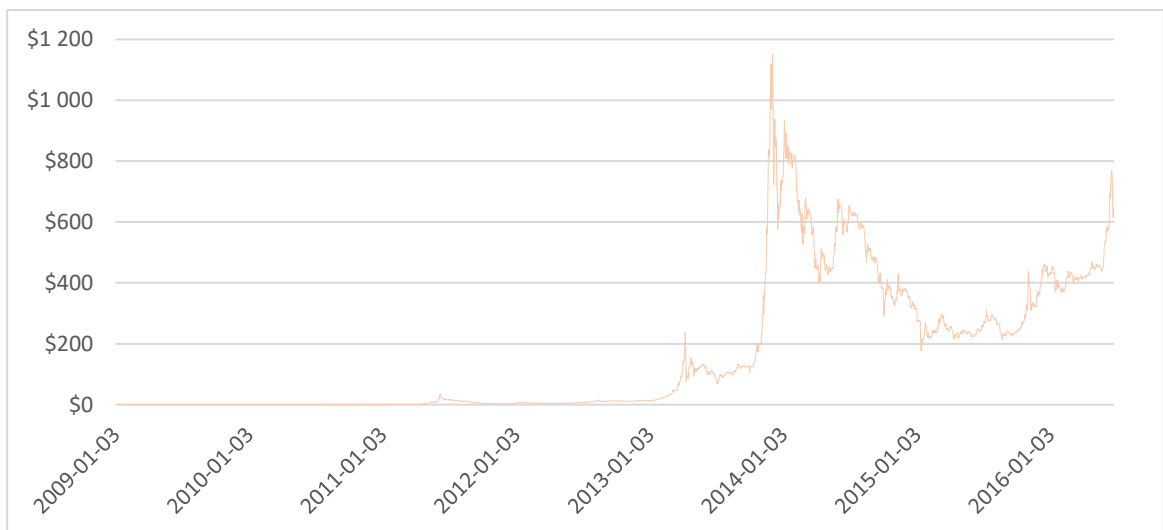


Source: Blockchain.info

Its market price has had several chapters of high volatility and market bubble behaviors, and has since January been steadily increasing, reaching a value of almost \$770 USD on the 16th July of 2016. This data is visible in figure 2 shown below.

Figure 2

Market Price USD



Source: Blockchain.info

The online query for the term ‘Bitcoin’ has decreased in periods near the middle of 2016, when compared to the levels from 2014, following the events of China’s Central Bank restrictions made the country biggest trading platform, BTC China, which stopped accepting deposits in yuan, as well as the crash of the biggest bitcoin exchange at the time, Mt. Gox, halting all customer withdrawals.

The need for more comprehensive studies on Bitcoin price evolution, its key drivers, as well as the study regarding the volatility of the price itself is self-evidentiary, given the global interest arising on these new types of financial assets. A broader understanding of the underlying mechanisms inherent to price formation allows for more prudent decisions by the holders of Bitcoin and for gains in market efficiency.

Literature Review

1 Literature on Money Theory

‘Currency’ is a term that has had a frequently mutable definition over the course of human history. A concept only existent under life as a society, since even the earliest forms of currency require an implicit expectation of the value of its form.

Presently, the Oxford English Dictionary defines a currency as “that which is current as a medium of exchange; the circulating medium (whether coins or notes)”. Its definition embodies the concept of circulation, regardless of its form, and the one of medium of exchange. Jevons (1875) summarily presents the functions of money in 4 points: medium of exchange; unit of count; standard of value and also a store of value.

As early as 9000 BC, cattle and grain were being used as stores of value and medium of exchange through bartering, according to Davies (2010). The utility and reliability of the things conferred them the value in trading, its acceptability. The use of metals as money was favored over commodities as cattle or salt, where available, given its durability, portability and divisibility. The concept of money is first mentioned in the *Book of Genesis*, and a metal currency already being used by the Philistine people around 1900 BC, as stated by Madden (1864).

With the later appearance of standardized minted coins, the value of the metal, and accordingly its weight, was guaranteed. However, the risk of manipulation of the value of the metal itself was present, given that the coins carried their facial value imprinted but still being subject to clipping in attempts to profit from recycling the precious metal.

Davies (2010) also affirms that later acceptance of other symbolic forms of money, such as tallies, bills of exchange or even banknotes, backed by the public trust in entities such as the Crown’s Treasury or goldsmiths, became a vital event for the creation of money beyond the natural thresholds of metal resources.

Paper money, originally introduced in China during the Song Dynasty during the 11th century, according to Headrick (2009), rooting from merchant receipts of deposits. The printing of larger amounts of money during the Mongol rule of the Yuan Dynasty,

following the Song Dynasty, due to a series of costly wars led to a spurt in inflation, as Ropp (2010) denotes.

Banknotes were introduced in Europe in the 13th century by intercontinental travelers such as William of Rubruck, or the more widely known Marco Polo, as he describes in his book *The Travels of Marco Polo*. Paper money presented an obvious advantage over coinage to merchants with regards to avoiding the physical burden of carrying substantial quantities of coins.

Banks later began to issue paper notes, aptly named ‘banknotes’, widely used as currency circulates today. The practice lasted until the end of the 17th century, in England, and still continued throughout the 19th century in the United States of America. At one point, over 5000 different banknotes issued by distinct banks were used, although only banknotes issued by the most creditworthy banks were commonly accepted, while the others tended to circulate locally. Banknotes issued by smaller and less known banks could be subject to acceptance at a discount rate, or not even accepted at all. The multiplying of types of money was enabled by the proliferation in the number of financial institutions.

The issuance of bank notes has since been replaced by government controlled and authorized banks. In 1694, the Bank of England was granted the sole rights for the creation of banknotes in England. The Federal Reserve was given similar rights after its establishment in 1913.

Thornton (1802) exposes the hypothesis of the ability of a central bank to control a currency’s price level adjusting the circulation through book keeping, thus enabling the central bank a command over a country’s money supply. The revelation of this idea set the foundation for the establishment of the quantitative theory of money.

Keynes (1924) presents an equation aiming to explain the influence of the money circulation amount and an ‘index for the cost of living’, as well as the proportion of currency the public held as circulation and as assets in bank deposits.

Equation 1

$$n = p(k + rk')$$

In his equation, n represented “currency notes or other forms of cash in circulation with the public”, p would represent “the index number of the cost of living” and r “the

proportion of the bank's potential liabilities (k') held in the form of cash". k and k' would represent, respectively, amounts "that the public, including the business world, finds it convenient to keep the equivalent of k consumption in cash and of a further available k' at their banks against cheques...".

His assumption, Keynes notes, that changes in n caused proportional changes in p could hold true in the long-term, but typically velocity, i.e. the number of times a unit of money is spent to buy goods and services per unit of time, and output were not stable in the short-term, and thus diminishing the relevance of money supply as a driver of general prices.

This theory was later reiterated by Milton Friedman (1956). While sharing similarities in views with Keynes, Friedman shifts back the focus of price drivers to the quantity of money.

Keynes's emphasized that the price level could be affected by changes in the product markets, such as induced by investment, and that changes in the money supply could affect output, and not merely prices, when in a scenario of an economy operating at less than full employment.

Friedman's restatement of the Quantitative Theory, a proposition that 'money matters' – i.e., that changes in the money supply could originate changes in nominal variables as well as in real ones, such as output and employment in the economy – limiting its main role to a theory on the demand for money.

Because money could be held by individuals not only as a store of value but also as a medium of exchange for financing transactions, it could be considered a good in terms of its real value, rather than its nominal one. This real value would be reduced by the rate of inflation, since the rate of inflation represents the cost of holding these real balances in place of holding commodities.

Two last points of divergence between Keynes' theory and Friedman's were the latter's assertion that the supply function of money was independent of the money demand function – some factors, like political and psychological ones – were important determinants of the money supply, according to Friedman; and that the function of the demand for money and the velocity of money were much more stable than originally theorized.

The equation of exchange, algebraically formulated by Fisher (1911), and more recently used in a simplified version by a considerable part of economists,

Equation 2

$$M \times V = P \times Q$$

relates M , the circulation plus deposits in checking and savings accounts held by the public; V , the velocity of money in final expenditures; with P , a price level, and Q , the real output, i.e., expenditure of an economy in macroeconomic equilibrium. The product of P and Q translates to the nominal value of money, and the finding of these variables is nowadays allowed due to developments and adoption of national income and product accounts.

The equation of exchange proposes that a causal effect can be drawn between M and P .

The development of the theories on money, under refinement even today, follows the theme that the value of currencies is dependent of the money in circulation and the velocity of the currency, among others already suggested above. These characteristics will be used in the pricing model presented later, as suggested by the literature exposed so far.

2 Literature on Bitcoin

The field of cryptofinance has only recently been started to be a part of economic and finance literature, so there are still some gaps available to address. An outstandingly large portion of the bitcoin research done so far relates to the areas of cryptography, computer science, security and systems design. In a 2015 report, the Committee of Payment and Market Infrastructures, from the Bank for International Settlements, highlights bitcoin's distributed ledger technology for the possibility of "making peer-to-peer payments in a decentralized network in the absence of trust between the parties or in any other third party", while also pointing out the possible lower costs to end users compared with existing centralized arrangements

In the field of social sciences there are currently three main branches of research being pursued: the first one deals mostly with the regulatory, legal and tax status of Bitcoin.

One of the questions being debated are the statutory nature of the Bitcoin, whether it should be considered an asset or a currency, being subject to distinct regulation and taxation in both cases. Glaser and Zimmerman (2014) make an inquiry to Bitcoin users in order to assess whether they consider BTC to be an alternative currency, or a speculative asset, concluding that especially the more uninformed users approaching digital currencies ‘are not primarily interested in an alternative transaction system but seek to participate in an alternative investment vehicle’.

In Germany, Bitcoin possesses a legal status as a ‘unit of account’, meaning that it can be used for tax and trading purposes in the country, something similar to ‘private money’, according to Clinch (2013); whereas in the US, Bitcoin is treated by the Internal Revenue System as ordinary income or as assets subject to capital gains taxes, depending on the circumstance, as stated by Drawbaugh and Temple-West (2014).

The aim of several papers written within this area is motivated by the need to reach some clarity on questions regarding options and procedures for users and investors, primarily in a shorter term. Debates on accounting procedures regarding Bitcoin have also been emerging, as we can see by the article by Raiborn and Sivitanides (2014), ‘Accounting Issues Related to Bitcoins’.

Another strand of research on Bitcoin emphasizes questions regarding fields such as sociology, anthropology, politics and even ethics surrounding the concept of Bitcoin. Angel and McCabe (2014) discuss the ethicality on the payment choices that employers provide their employees on the form of the payment; concluding that payments through bitcoin, although fairly recent, aren’t good or evil on their own, but it is the ethicalness behind the use of the payment system itself that matters.

Lastly, the third area focuses on models creation to formulate representations: evaluate price movements, changes on fundamentals or analyze incentive structures for Bitcoin miners, as well as sustainability of low transaction fees. On the following paragraphs we will discuss in deeper detail the conclusions established on this field since the beginning of the Bitcoin project.

Bitcoin has its origins, however, in a paper by author Satoshi Nakamoto (2008), as a decentralized electronic cash system whose complete list of transactions is publicly available. Since the establishment of its genesis block was the 3rd of January 2009 and the announcement of the project on the Cryptography mailing list on January 11th 2009,

and at the time of the writing of this piece, there are approximately BTC 15,698,000 in circulation. This dominance of Bitcoin over other existing crypto currencies is discussed by Bornholdt & Sneppen (2014), whose model designed through voter-like dynamics demonstrates that Bitcoin shows no particular characteristic over other crypto currencies, allowing for the possibility being replaced by a competing crypto currency.

Kondor et al (2014) take advantage of the publicity of all monetary exchanges of coins, which provides unprecedented opportunity to study monetary transactions of individuals, validating the assumption that the growth of the whole network is related to the greater acceptance of Bitcoin as a method of payment. It is further shown in their work that the wealth in bitcoins is accumulating in time and that such accumulation is tightly related to the ability to attract new connections in the network.

In regards to the focal point of the thesis to be developed, the evolution of price, its key drivers and the volatility of the price, a few studies reveal interesting conclusions that can narrow down the scope of the analysis we intend to develop. Buchholz et al, 2012, run several ARCH/GARCH models which display that, before the peak of the “first” market bubble of Bitcoin during the summer of 2011, there are asymmetrical effects to positive and negative shocks.

Fink and Johann (2014) give a detailed insight into the market microstructure of Bitcoin based on the more detailed pricing data available at the time of their work. Their interesting conclusions can constitute the basis for further assumptions on Bitcoin market participants and the market behavior. The study supports the idea that the price is not informationally efficient, that speculation and non-fundamental price movements seem to be dominant. A cross exchanges analysis also shows an improvement of liquidity over Bitcoin’s lifespan, with clear indicators between prices and the liquidity on the different exchanges. The authors also provide a picture of the market structure, asserting that Bitcoin is traded by both retail and professional traders, employing different strategies, and that a large fraction of all Bitcoin outstanding and transaction volume is generated by only a few market participants, which the authors hypothesize being market exchanges, investment funds or mining firms.

Opening a new line of inquiry into Bitcoin’s price formation based on a possible proxy for the asset demand, Kristoufek (2013) analyzes the dynamic relationship between the

Bitcoin price and the interest in the currency measured by search queries on Google Trends and frequency of visits on the Wikipedia page on Bitcoin.

The conclusions show that, apart from a very strong correlation between price level of the digital currency and both the Internet engines, a strong causal relationships between the prices and searched terms is also found. This relationship is in fact found to be bidirectional, which the author finds expectable about a financial asset with no underlying fundamentals.

Specifically, the queries are found to be pro-cyclical in relation to the price, with the increasing interest pushes the prices further atop when the prices were already considered high; while, if the prices were below their trend, the growing interest pushed the prices even deeper, forming “an environment suitable for a quite frequent emergence of a bubble behavior which indeed has been observed for the Bitcoin currency”.

Garcia et al., 2014, also address the question of Bitcoin market bubbles using digital behavioral traces of investors in their social media use, search queries and user base. They find positive feedback loops for social media use and the user base, reinforcing the conclusions stated in the previous work exposed.

Viglione (2015) takes a different approach to price determinants, demonstrating an inverse relationship between economic freedom and bitcoin price premiums. Viglione states that economic freedom, i.e., controls on capital circulation and foreign exchange, have shown to increase the premium investors pay over global prices, already accounting for market microstructure differences, such as trading volume and bid-ask spreads.

Since Bitcoin offers an efficient way to diversify financial assets internationally with minimal transaction costs, investors in countries with higher resource confiscation via taxation have been found willing to pay more for it than investors in lower tax countries.

Lastly, Kristoufek (2015) finds that despite some considerations about the speculative nature of Bitcoin, its price in the long term is found to be related with standard fundamental factors, such as usage in trade, money supply and price level, following the general monetary economic theories.

Methodology

This chapter intends to briefly describe the methodology of the analysis presented in the following section. As previously stated, literature indicates that a relevant driver of Bitcoin price is the interaction between demand and supply (Bartos, J., 2015; Kristoufek L., 2015). Other factors that have been found to affect the price are the information provided by social media and the sentiment conveyed in related articles; the transactional needs of users (Polasik, M., 2014); and also the price level of Bitcoin itself (Kristoufek L., 2015).

1 Methodological aspects

First we make use of the data to construct a model of the Bitcoin price based on the remaining variables through a vector error correction model, VEC, after testing the time series for cointegration. The resulting equation yields conclusions on the effect, in percentage, on the Bitcoin price from a 1% variation in each variable. It also states if and how the different variables are affected by deviations from the bitcoin price long-run equilibrium.

Subsequently, we fit heteroscedastic models, given the existence of volatility clusters in the series, which can hint that variability may evolve over time. The construction of these models, namely the GARCH-in-mean, the exponential GARCH-in-mean and the Threshold GARCH-in-mean, is suggested by the existence of conditional heteroscedasticity.

2 Sample

The time series used is composed of the following variables: Bitcoin price; Standard & Poor's 500 index (*SP500*); daily treasury real yield curve rates on TIPS - “Treasury Inflation Protected Securities” for a fixed maturity of 7 years (*TIPS7*); daily USD price per ounce of gold (*Gold*); daily number of confirmed bitcoin transactions (*Number of transactions*); total number of unique addresses used on the Bitcoin blockchain (*Unique*

Addresses); total value of coinbase block rewards and transaction fees paid to miners (*Miner Fees*) and daily number of the term ‘Bitcoin’ queries made in Wikipedia (*Wiki Queries*).

Data for the *SP500*, *Gold*, *Number of transactions*, *Unique Addresses* and *Miner Fees* were all obtained through Quandle (www.quandl.com), *TIPS7* was obtained from the U.S. Department of Treasury (www.treasury.gov) and *Wiki Queries* was obtained from the Wikipedia article traffic statistics (<http://stats.grok.se>).

Additional information on the data, such as the number of observations and its timespan can be found in the following chapter.

Empirical Analysis

The time series spans the period from November 1, 2013 to January 20, 2016, a sample size of 553 observations.

Because some of the values in the TIPS series are negative, we considered $(1 + \text{TIPS}_t)$ which is positive, just like a gross return time series. The data are in logs form (the natural logarithm is denoted by \log).

3 Error correction model

3.1 Stationarity

The plots in Figure 3 and the correlograms in Figure 4 both show that the time series are all nonstationary. In fact, as shown in the latter figure, the sample autocorrelations are large and decay very slowly, being significant for a lag as large as 30, thus proving nonstationarity.

Figure 3
Time series plots (levels)

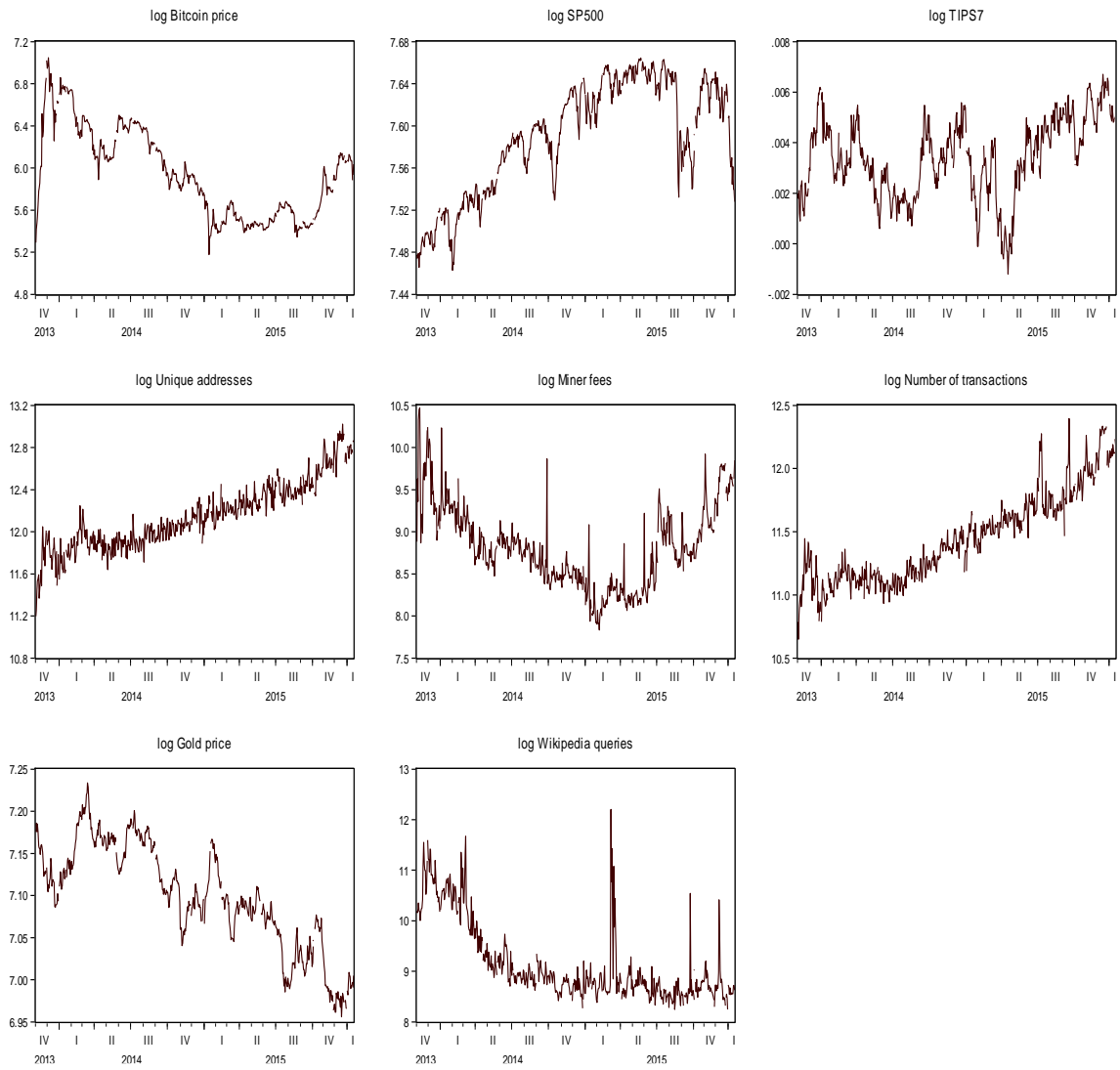
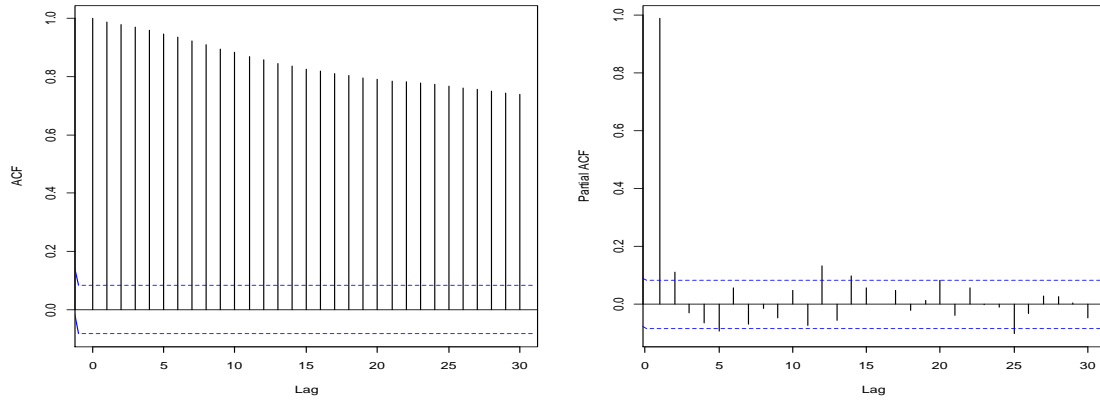
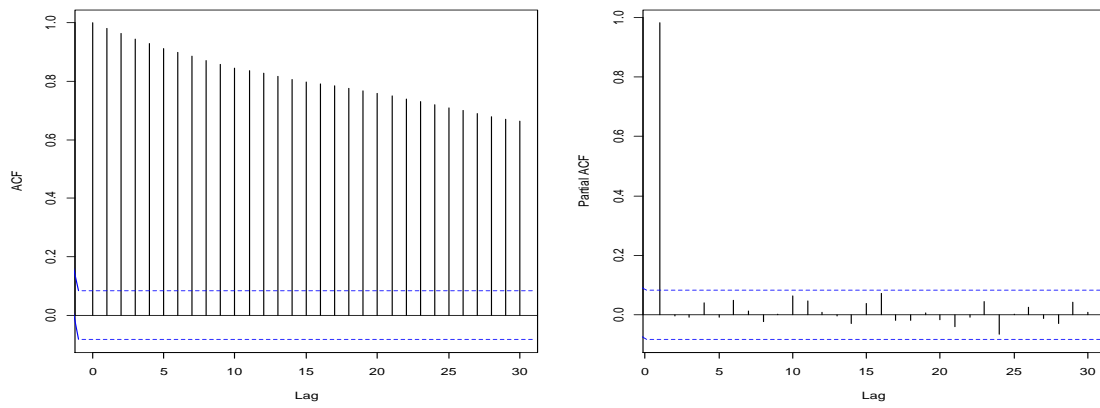


Figure 4
Correlograms (levels)

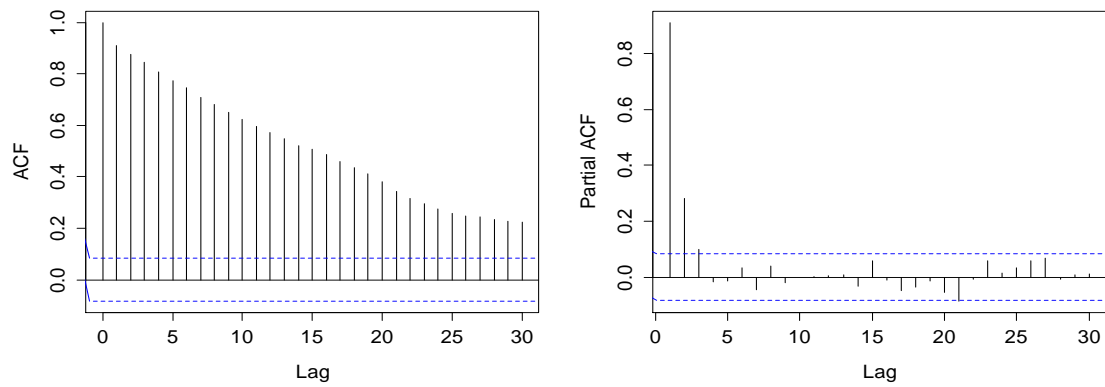
log Bitcoin price



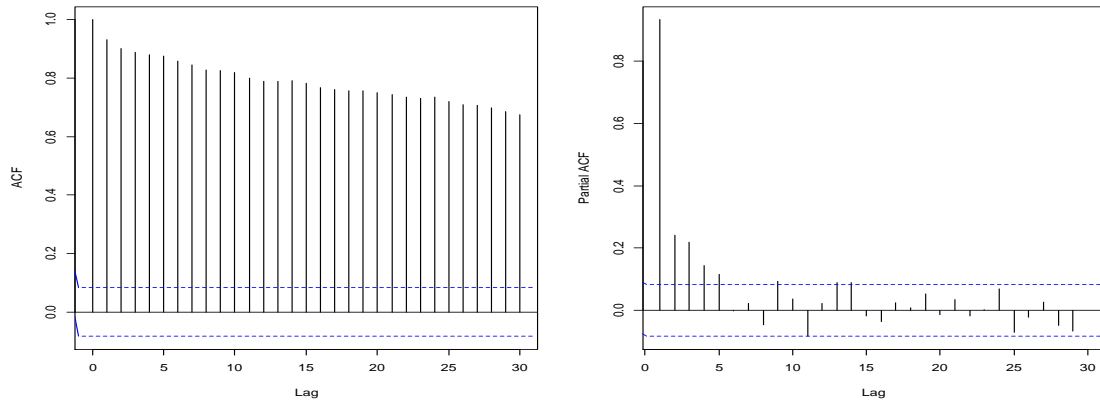
log SP500



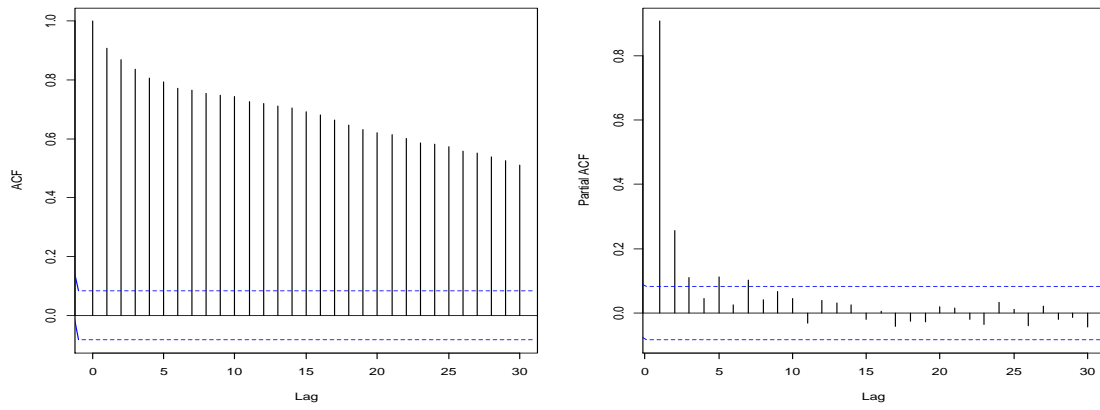
log TIPS



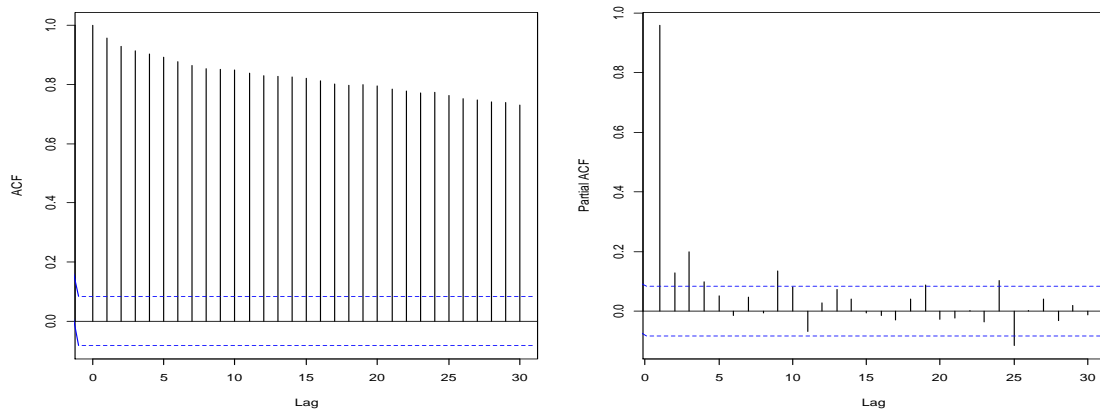
log Unique addresses



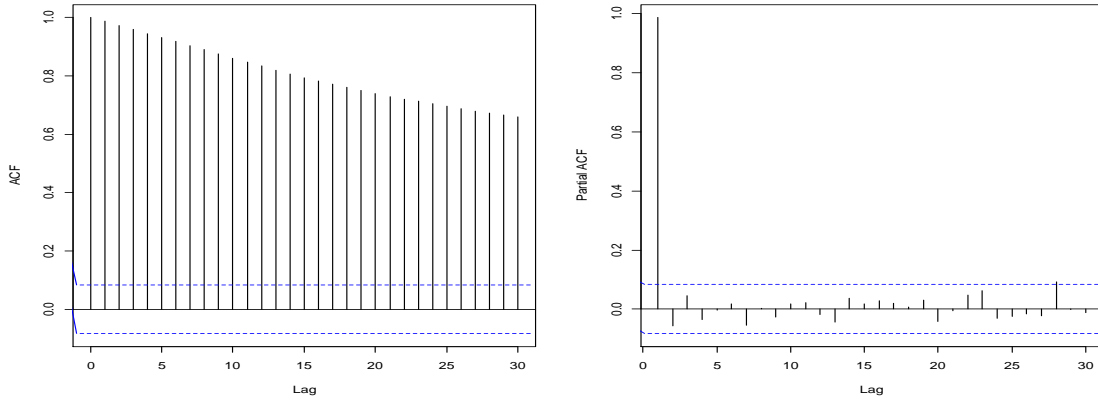
log Miner fees



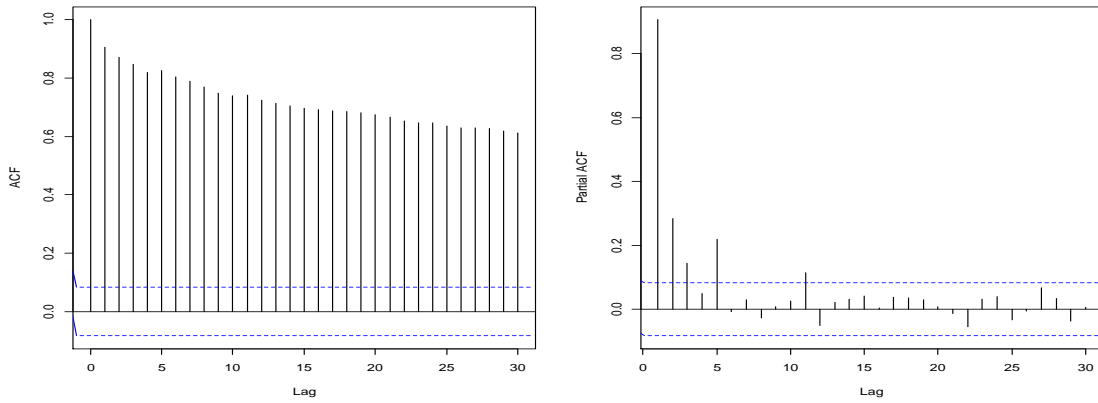
log Number of transactions



log Gold price



log Wikipedia queries



Furthermore, unit root tests were also run: ADF (Dickey and Fuller, 1979), DF-GLS (Elliot, Rothenberg and Stock, 1996) and KPSS (Kwiatkowski, Phillips, Schmidt and Shin, 1992) tests.

Concerning the ADF test for the series in levels, the model with a constant was chosen to take a nonzero mean into account, except in the case of the TIPS, where the model without a constant was adopted because it was not required (the mean was approximately zero). The model order was selected by the sample correlogram of the differenced series, by the AIC, the Schwarz and the Hannan-Quinn criteria and by analyzing the statistical significance of the model estimated parameters. Table 1 displays the model order (where $p = 0$ denotes the DF test), the test statistic and the MacKinnon (1996) critical points (5% significance level) and p-values for the level series and for the differenced series (for which the model without a constant was chosen). The null hypothesis of a unit root could not be rejected for all the series in

levels but it was rejected for the differenced time series ($d = 1$). Therefore, they are nonstationary in levels but are stationary after a single difference, i.e., they are integrated of order 1 or $I(1)$.

Table 1
ADF test

Time series	Level series ($d = 0$)				Differenced series ($d = 1$)			
	p	Test stat.	Critical point	p-value	p	Test stat.	Critical point	p-value
Bitcoin price	1	-1.74	-2.87	0.41	0	-25.88	-1.94	0.00
SP500	0	-2.33	-2.87	0.16	0	-22.92	-1.94	0.00
TIPS	1	-0.23	-1.94	0.26	0	-27.88	-1.94	0.00
Gold price	6	-1.63	-2.87	0.47	5	-8.64	-1.94	0.00
Number transactions	3	-1.56	-2.87	0.50	2	-18.84	-1.94	0.00
Unique addresses	4	-1.52	-2.87	0.52	3	-18.66	-1.94	0.00
Miner fees	6	-2.40	-2.87	0.14	5	-14.19	-1.94	0.00
Wikipedia queries	4	-2.48	-2.87	0.12	3	-17.92	-1.94	0.00

The DF-GLS test results are displayed in table 2. The model with a constant was chosen for the series in levels and recall that the MacKinnon (1996) critical points are used. The null hypothesis of a unit root was not rejected for all the time series in levels except for the miner fees and the TIPS. Therefore, the former was considered stationary by this test. Concerning the latter, since the null hypothesis could not be rejected at the 1% level (the critical point is -2.57), a difference was considered. The unit root hypothesis was rejected for the differenced time series ($d = 1$). Thus, this test concluded that the time series are $I(1)$ except the miner fees which is stationary. Consequently, a different conclusion was reached by this test relatively to the ADF test concerning the miner fees but the conclusions were coincident for all the other time series.

Table 2
DF-GLS test

Time series	Level series (d = 0)			Differenced series (d = 1)		
	p	Test stat.	Critical point	p	Test stat.	Critical point
Bitcoin price	1	-0.70	-1.94	0	-8.32	-1.94
SP500	0	-0.82	-1.94	0	-20.59	-1.94
TIPS	1	-2.35	-1.94	0	-24.31	-1.94
Gold price	0	-0.43	-1.94	0	-12.32	-1.94
Number transactions	3	0.13	-1.94	2	-4.77	-1.94
Unique addresses	4	0.89	-1.94	3	-8.39	-1.94
Miner fees	4	-2.78	-1.94			
Wikipedia queries	4	-1.18	-1.94	3	-17.31	-1.94

The KPSS test results are displayed in table 3. The model with a constant and a trend was chosen for all the time series except the TIPS for which the model with a constant only was considered. The truncation lag ℓ in the test statistic was set equal to 6, according to the usual rule given by $\ell = 4(n/100)^{1/4}$ where n is the sample size. The critical points are those given in Kwiatkowski, Phillips, Schmidt and Shin (1992). The null hypothesis of stationarity (no unit roots) was rejected for all the time series in levels and it was not rejected for the differenced time series ($d = 1$), leading to the conclusion that they are $I(1)$.

Table 3
KPSS test

Time series	Level series (d = 0)		Level series (d = 1)	
	Test stat.	Critical point	Test stat.	Critical point
Bitcoin price	0.853	0.146	0.207	0.463
SP500	1.251	0.146	0.255	0.463
TIPS	1.419	0.463	0.024	0.463
Gold price	0.559	0.146	0.029	0.463
Number transactions	0.857	0.146	0.015	0.463
Unique addresses	0.602	0.146	0.043	0.463
Miner fees	1.550	0.146	0.083	0.463
Wikipedia queries	1.124	0.146	0.020	0.463

The three tests generally agreed on the stationarity of the time series with the only exception of the ADF-GLS test concerning the miner fees. Based on the test results, we could conclude that the time series are integrated of order 1. The plots and the correlograms of the differenced series shown in figures 5 and 6 respectively confirmed this conclusion since the former shows no trend and the sample autocorrelations and partial autocorrelations cut off or tail off in the first few lags.

Figure 5
Time series plots (differenced series)

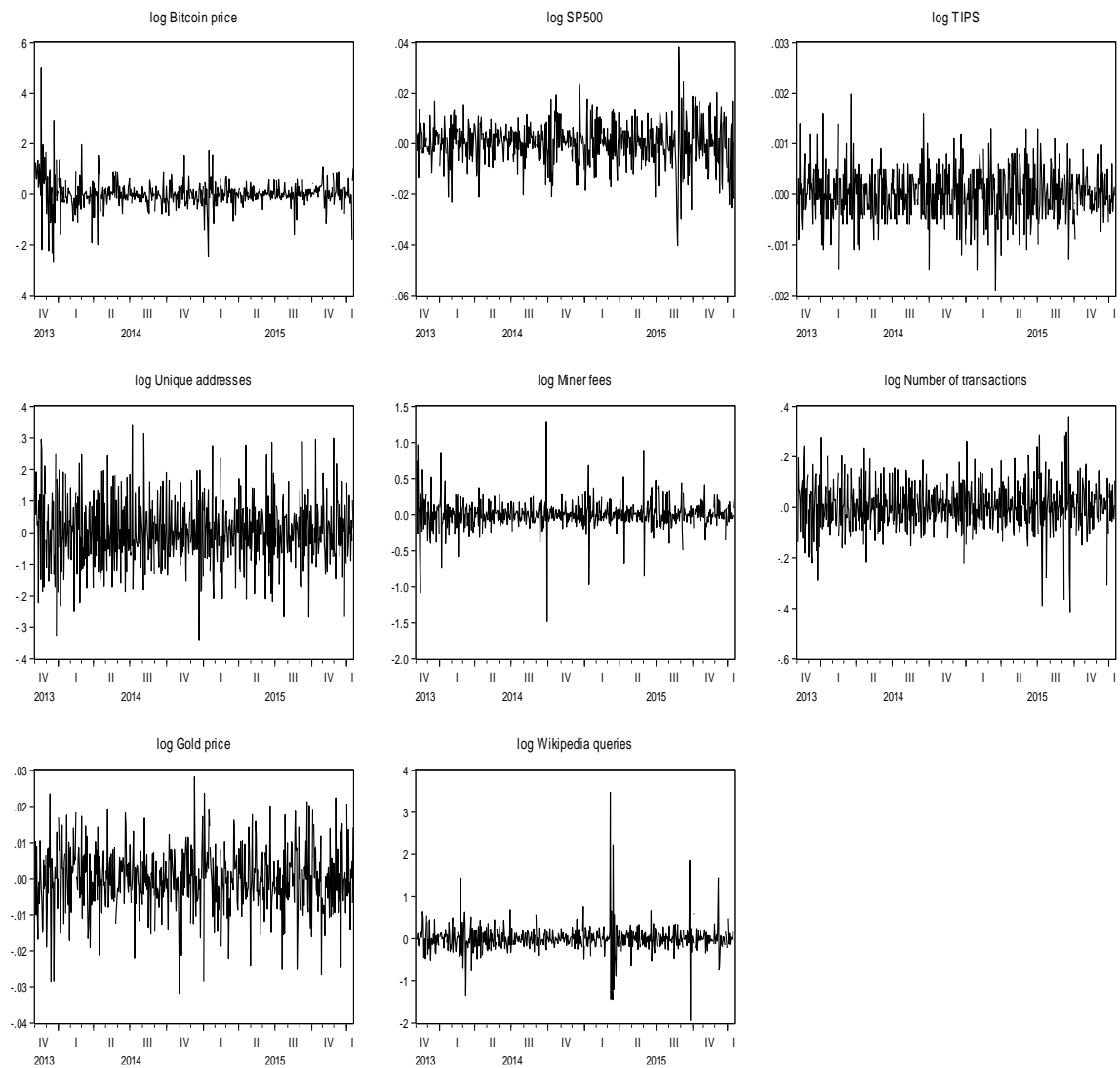
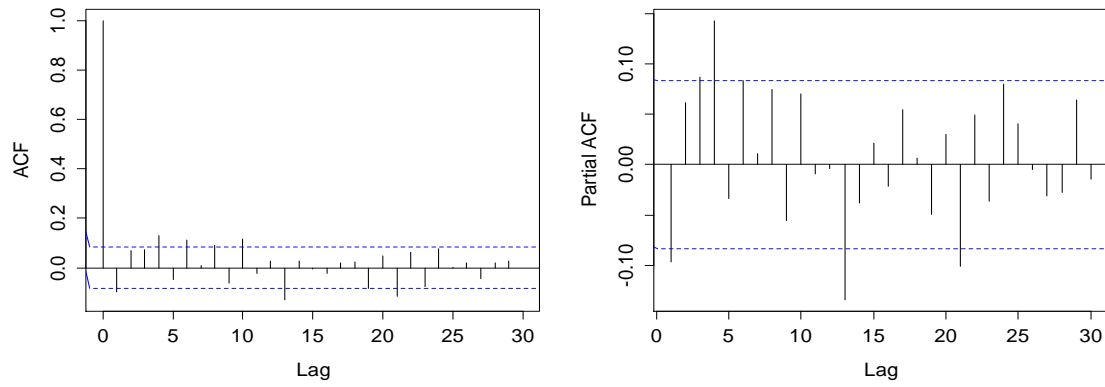
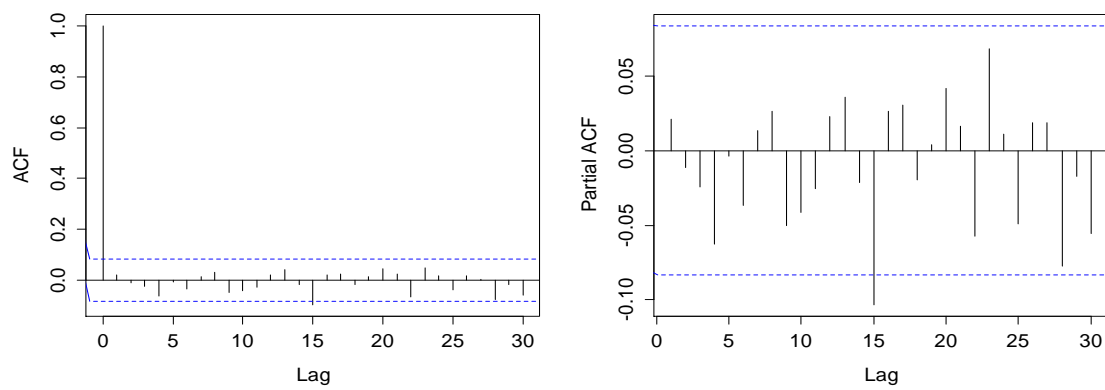


Figure 6
Correlograms (differenced series)

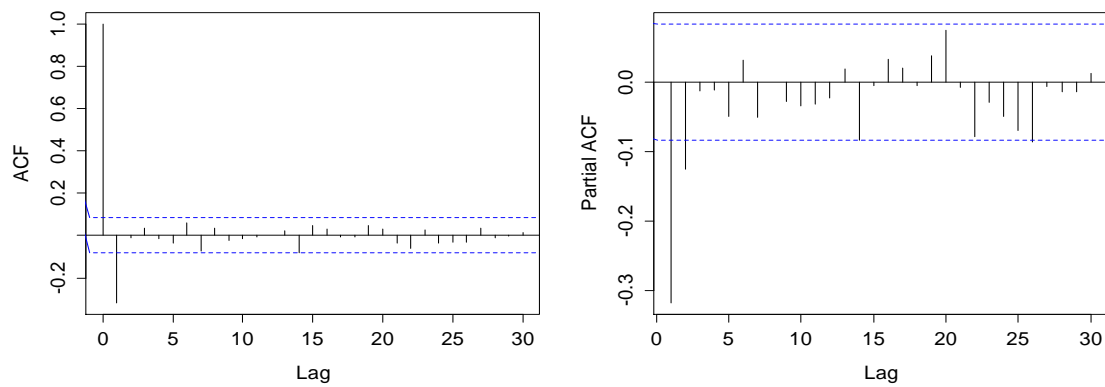
log Bitcoin price



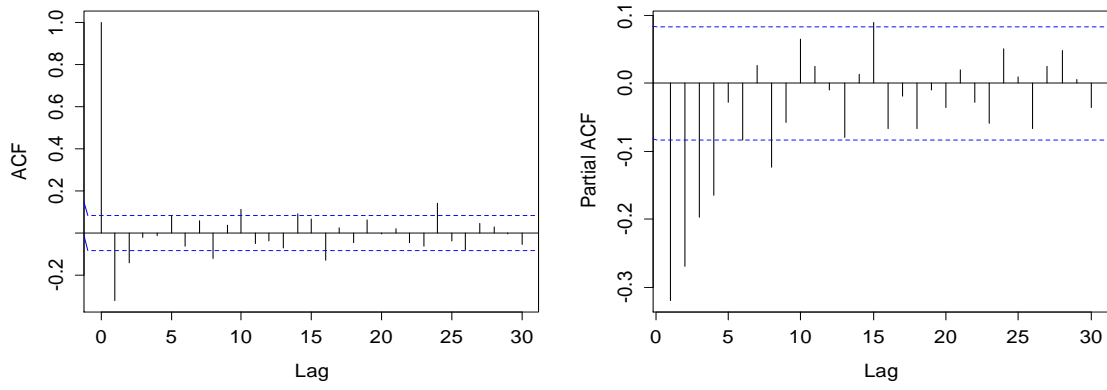
log SP500



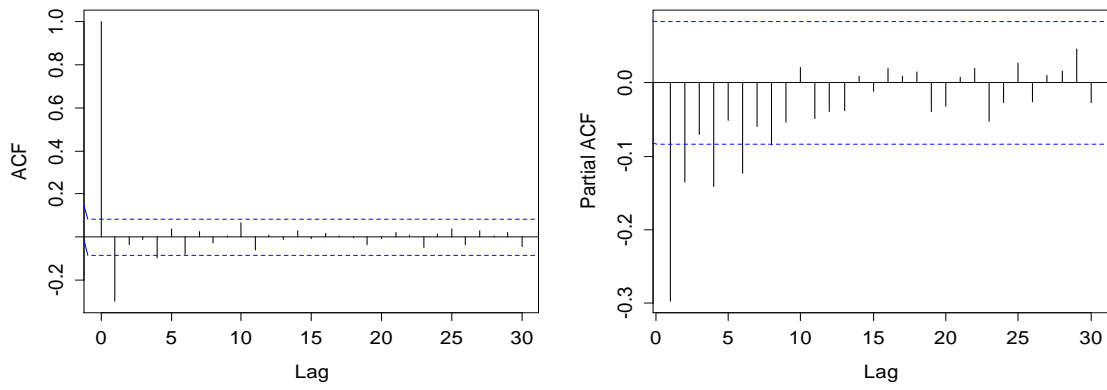
log TIPS



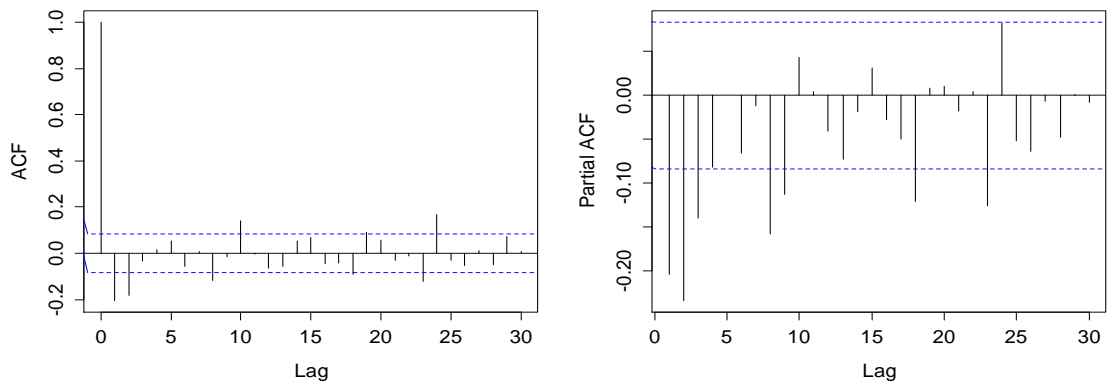
log Unique addresses



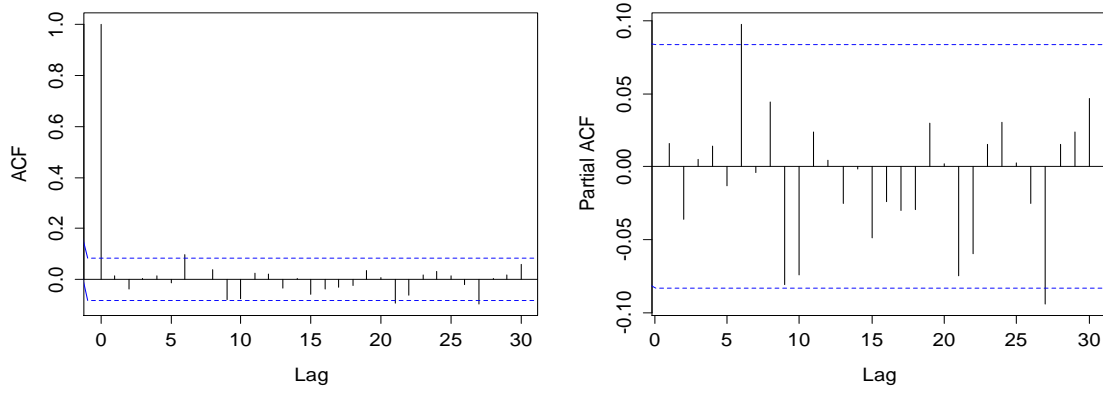
log Miner fees



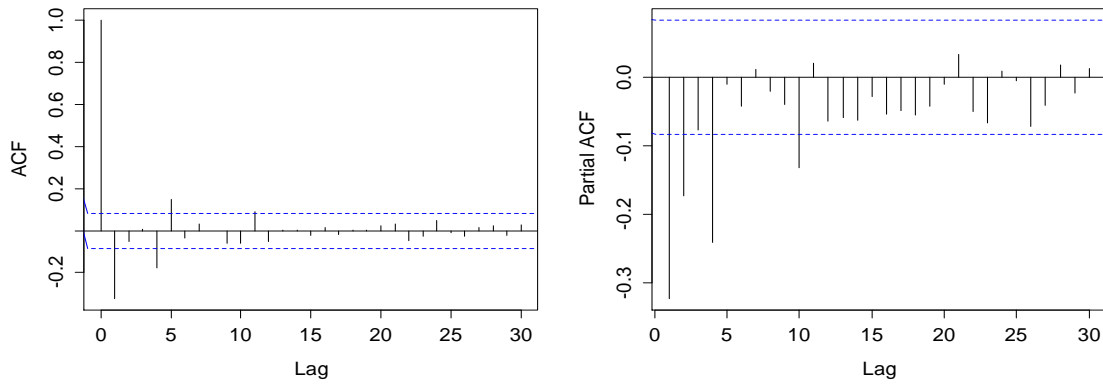
log Number of transactions



log Gold price



log Wikipedia queries



3.2 Cointegration

Since the time series were $I(1)$, the next step was then to investigate the existence of cointegration among them. If they were cointegrated, a Vector error-correction model (VECM) could be fit and a long-run equation for the bitcoin price could be estimated. To test for cointegration, Johansen likelihood-ratio tests were used (Johansen, 1988, 1991; Johansen and Juselius, 1990), based on a VECM with a constant in the cointegrating equation. The model order p was selected by the AIC, the Schwarz and the Hannan-Quinn criteria and by analyzing the statistical significance of the model estimated autoregressive parameter matrices which led to $p = 3$. Since there are 8 variables, the maximum number of (linearly independent) cointegrating vectors is 7. Tables 4 and 5 show the results of the trace and of the maximum eigenvalue tests

respectively where the critical points and the p-values are given by Mackinnon, Haug and Michaelis (1999). The former test led to a cointegrating rank $r = 3$, since it was the lowest rank for which the null hypothesis could not be rejected at the 5% level but, at the 1% level, the cointegrating rank would be $r = 2$ (with a p-value of 0.03). The latter test led to a cointegrating rank $r = 2$, since it was the lowest rank for which the null hypothesis could not be rejected. Therefore, we concluded that the variables are cointegrated with a cointegrating rank $r = 2$ and a suitable VEC model could then be fitted.

Table 4
Trace test

Cointegrating rank	Eigenvalue	Test stat.	Critical point	p-value
$r = 0$	0.1769	291.05	169.60	0.00
$r \leq 1$	0.1293	184.19	134.68	0.00
$r \leq 2$	0.0611	108.16	103.85	0.03
$r \leq 3$	0.0496	73.55	76.97	0.09
$r \leq 4$	0.0368	45.61	54.08	0.23
$r \leq 5$	0.0277	25.00	35.19	0.40
$r \leq 6$	0.0109	9.56	20.26	0.68
$r \leq 7$	0.0064	3.54	9.17	0.49

Table 5
Maximum eigenvalue test

Cointegrating rank	Eigenvalue	Test stat.	Critical point	p-value
$r = 0$	0.1769	106.85	53.19	0.00
$r \leq 1$	0.1293	76.04	47.08	0.00
$r \leq 2$	0.0611	34.61	40.96	0.22
$r \leq 3$	0.0496	27.94	34.81	0.26
$r \leq 4$	0.0368	20.61	28.59	0.37
$r \leq 5$	0.0277	15.44	22.30	0.34
$r \leq 6$	0.0109	6.02	15.89	0.79
$r \leq 7$	0.0064	3.54	9.16	0.49

3.3 Vector Error Correction Model

The VEC model order was selected by the AIC, the Schwarz and the Hannan-Quinn criteria and by analyzing the statistical significance of the estimated (short-term) autoregressive parameters which led to an order $p = 3$. Therefore, the VEC(3) model was estimated by maximum likelihood:

Equation 3

$$\begin{pmatrix} \nabla \log(\text{BTCP})_t \\ \nabla \log(\text{SP500})_t \\ \nabla \log(\text{TIPS})_t \\ \nabla \log(\text{UA})_t \\ \nabla \log(\text{MF})_t \\ \nabla \log(\text{NTR})_t \\ \nabla \log(\text{GOLD})_t \\ \nabla \log(\text{WIKI})_t \end{pmatrix} = \hat{\alpha} \hat{\beta}^T \begin{pmatrix} \log(\text{BTCP})_{t-1} \\ \log(\text{SP500})_{t-1} \\ \log(\text{TIPS})_{t-1} \\ \log(\text{UA})_{t-1} \\ \log(\text{MF})_{t-1} \\ \log(\text{NTR})_{t-1} \\ \log(\text{GOLD})_{t-1} \\ \log(\text{WIKI})_{t-1} \\ 1 \end{pmatrix} + \sum_{i=1}^3 \hat{\Phi}_i \begin{pmatrix} \nabla \log(\text{BTCP})_{t-i} \\ \nabla \log(\text{SP500})_{t-i} \\ \nabla \log(\text{TIPS})_{t-i} \\ \nabla \log(\text{UA})_{t-i} \\ \nabla \log(\text{MF})_{t-i} \\ \nabla \log(\text{NTR})_{t-i} \\ \nabla \log(\text{GOLD})_{t-i} \\ \nabla \log(\text{WIKI})_{t-i} \end{pmatrix}$$

where BTCP, SP500, TIPS, UA, MF, NTR, GOLD and WIKI denote respectively the bitcoin price, the SP500 index, the TIPS, the unique addresses, the miner fees, the number of transactions, the gold price and the Wikipedia queries and the estimated loading and cointegrating matrices are respectively (asymptotic standard errors within parentheses)

Equation 4

$$\hat{\alpha} = \begin{pmatrix} -0.038 & 0.077 \\ (0.013) & (0.019) \\ -2 \times 10^{-5} & 4 \times 10^{-4} \\ (0.002) & (0.003) \\ -2 \times 10^{-4} & 2 \times 10^{-4} \\ (1 \times 10^{-4}) & (2 \times 10^{-4}) \\ 0.038 & 0.068 \\ (0.021) & (0.031) \\ 0.208 & -0.322 \\ (0.040) & (0.059) \\ -0.039 & -0.031 \\ (0.020) & (0.029) \\ 0.002 & -0.005 \\ (0.002) & (0.003) \\ 0.090 & 0.112 \\ (0.068) & (0.102) \end{pmatrix}; \quad \hat{\beta} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 58.598 & 7.102 \\ (21.762) & (15.058) \\ -3.108 & -2.185 \\ (0.369) & (0.255) \\ -0.734 & 0.165 \\ (0.069) & (0.048) \\ 3.582 & 1.915 \\ (0.348) & (0.241) \\ 1.923 & 1.849 \\ (0.848) & (0.587) \\ -0.110 & -0.095 \\ (0.049) & (0.034) \\ -15.579 & -16.680 \\ (7.180) & (4.968) \end{pmatrix}.$$

Note that the first two rows of $\hat{\beta}$ form a (2×2) identity matrix because of the required parameter identifying restrictions. As a consequence, the SP500 was left out of the Bitcoin price cointegrating equation but that was not relevant. In fact, a VEC model was tried with a single cointegrating equation with the bitcoin price as the dependent variable and all the others as independent, but the estimate of the parameter associated with the SP500 was not statistically significant which means that this variable should not be included in the equation of the bitcoin price. Note also that the parameter of the bitcoin price in the first equation was normalized to be 1, defining an equation with this variable as dependent which was our main purpose in this analysis. Furthermore, the estimated short-term autoregressive parameter matrices $\hat{\Phi}_1, \hat{\Phi}_2, \hat{\Phi}_3$ are left to the appendix because of their large dimension.

From the VEC(3) model above, the first cointegration equation is

Equation 5

$$\hat{\log(\text{BTCP})}_t = 15.579 - 58.598 \log(\text{TIPS})_t + 3.108 \log(\text{UA})_t + 0.734 \log(\text{MF})_t \\ - 3.582 \log(\text{NTR})_t - 1.923 \log(\text{GOLD})_t + 0.110 \log(\text{WIKI})_t$$

or, in terms of the original variables,

Equation 6

$$\hat{\text{BTC}}_t = e^{15.579} \text{TIPS}_t^{-58.598} \text{UA}_t^{3.108} \text{MF}_t^{0.734} \text{NTR}_t^{-3.582} \text{GOLD}_t^{-1.923} \text{WIKI}_t^{0.110}$$

where the parameters are interpreted as elasticities. The normalized parameters with the identifying restrictions and the appropriate standard errors made inference possible. Thus, t-statistics could be computed to test the significance of the estimated parameters showing that they are all statistically significant (5% significance level). Consequently, we concluded that these variables (except the SP500) affect the bitcoin price.

Accordingly, the parameter estimates could be interpreted:

- A 1% increase (decrease) of the TIPS causes an estimated 58.6% decrease (increase) of the bitcoin price with all the other variables constant.
- A 1% increase (decrease) of the unique addresses causes an estimated 3.1% increase (decrease) of the bitcoin price with all the other variables constant.
- A 1% increase (decrease) of the miner fees causes an estimated 0.7% increase (decrease) of the bitcoin price with all the other variables constant.
- A 1% increase (decrease) of the number of transactions causes an estimated 3.6% decrease (increase) of the bitcoin price with all the other variables constant.
- A 1% increase (decrease) of the gold price causes an estimated 1.9% decrease (increase) of the bitcoin price with all the other variables constant.
- A 1% increase (decrease) of the Wikipedia queries causes an estimated 0.11% increase (decrease) of the bitcoin price with all the other variables constant.

Therefore, the bitcoin price responds positively to the unique addresses, the miner fees and the Wikipedia queries and responds negatively to the TIPS, the number of transactions and the gold price. The TIPS have the strongest effect, followed by the number of transactions, unique addresses, the gold price, miner fees and Wikipedia queries (SP500 has no effect on bitcoin price).

Concerning the loading matrix α , the relevant elements for this analysis are those in the first column. The loading estimates in the equations of the bitcoin price, of the miner fees and of the number of transactions are significant at the 5% level, whereas the loading in the equation of the unique addresses is only significant at the 10% level:

- The estimated loading in the equation of the bitcoin price shows that a positive (negative) deviation of the (log) bitcoin price relatively to the long-run equilibrium induces a decrease (increase) in itself equal to 0.038 in the short run, i.e., an over-priced bitcoin causes the price to decrease and vice-versa (as expected, the sign of this loading is negative). This shows that price adjustments towards the equilibrium caused by a price deviation from its equilibrium value are made at the speed of 0.038 (for the logs).
- The estimated loading in the equation of the unique addresses shows that a positive (negative) deviation of the (log) bitcoin price relatively to the long-run equilibrium induces an increase (decrease) of the (log) unique addresses equal to 0.038 in the short run, i.e., an over-priced bitcoin causes the unique addresses to increase and vice-versa. This shows that unique addresses adjustments towards the equilibrium caused by a price deviation from its equilibrium value are made at the speed of 0.038 (for the logs). Note that this estimated loading is significant at the 10% level only which means that the effect is weaker than for the other three variables.
- The estimated loading in the equation of the miner fees shows that a positive (negative) deviation of the (log) bitcoin price relatively to the long-run equilibrium induces a decrease (increase) of the (log) miner fees equal to 0.208 in the short run, i.e., an over-priced bitcoin causes the miner fees to decrease and vice-versa. This

shows that miner fees adjustments towards the equilibrium caused by a price deviation from its equilibrium value are made at the speed of 0.208 (for the logs).

- The estimated loading in the equation of the number of transactions shows that a positive (negative) deviation of the (log) bitcoin price relatively to the long-run equilibrium induces a decrease (increase) of the (log) number of transactions equal to 0.039 in the short run, i.e., an over-priced bitcoin causes the number of transactions to decrease and vice-versa. This shows that the number of transactions adjustments towards the equilibrium caused by a price deviation from its equilibrium value are made at the speed of 0.039 (for the logs).

The remaining variables are not significantly affected by the deviation from the bitcoin price long-run equilibrium. Therefore, only variables directly involved in the bitcoin trade (price, unique addresses, miner fees, number of transactions) are affected by such a disequilibrium error (the Wikipedia queries are the only exception).

Finally, we note that, since our purpose was to study the bitcoin price formation, the second cointegration equation and consequently the second column of the loading matrix were not analyzed because they were not relevant.

4 Heteroscedastic Models

The plots of the differenced time series in Figure 3 show that the existence of volatility clusters, i.e., volatility is high in certain time periods and low in others meaning that variability may evolve over time. Therefore, taking into account the effect of volatility on the bitcoin price may be necessary. To this purpose, heteroscedastic models are appropriate, such as the autoregressive conditional heteroscedastic model (ARCH) of Engle (1982) and the generalized autoregressive conditional heteroscedastic model (GARCH) of Bollerslev (1986).

4.1 Univariate Analysis

We first tested each (differenced) time series for conditional heteroscedasticity. The fitted models were identified from the sample ACF and PACF (Figure 4) and from the AIC and Schwarz selection criteria. Diagnostic checking was also performed in order to find the best model.

- Bitcoin price – The sample ACF and PACF of the differenced log Bitcoin price both tail off (Figure 4), suggesting an ARMA model. We tried the usual ARMA(p,q) models with $1 \leq p, q \leq 2$ and the best fitted model was the ARMA(2,1) (standard errors in parentheses)

Equation 7

$$\begin{array}{c} (1 - 0.683B - 0.177B^2) \nabla \log(\text{BTCP})_t = (1 + 0.789B) \hat{a}_{1t} \\ (0.091) \quad (0.043) \qquad \qquad \qquad (0.084) \end{array}$$

where \hat{a}_{1t} denotes the residuals and B is the backshift operator such that $B^j X_t = X_{t-j}$. The estimated parameters are significant and verify the stationarity and invertibility conditions. The residuals are plotted in Figure 5 and show the volatility clusters already exhibited by the differenced log Bitcoin price. In order to test for heteroscedasticity, the residual ACF and PACF are displayed in Figure 6 and both show only two significant values at large lags with no relevance. These lags

cause the Ljung-Box statistic to be significant with a value of 49.4 and a p-value of 0.014 for 30 lags. However, since these significant ACF values occur at lags with no relevance, we ignored the significance of the Ljung-Box statistic (and note that it is nonsignificant at the 1% level) and did not reject the hypothesis that \hat{a}_{1t} is a white noise.

On the contrary, the sample ACF and PACF of the squared residuals (Figure 7) show many significant values, suggesting the existence of conditional heteroscedasticity (Tsay, 2010). Furthermore, the Ljung-Box statistic for 30 lags is 162.4 with a p-value of approximately 0, i.e., highly significant (and it is significant for all the lags considered) and the Lagrange-Multiplier test statistic (Engle, 1982) is 191.3 (30 lags) with a p-value of approximately 0. Thus, the presence of conditional heteroscedasticity (ARCH effect) is confirmed. Since the sample ACF and PACF of the squared residuals both tail off, a GARCH model appeared to be appropriate. We started with a GARCH(1,1) and the joint estimation of an ARMA(2,1)-GARCH(1,1) led to (standard errors in parentheses)

Equation 8

$$\begin{aligned} (1 + 0.669B + 0.0004B^2) \nabla \log(\text{BTCP})_t &= (1 - 0.703B) \hat{a}_{1t} \\ (0.319) \quad (0.027) & \quad (0.311) \\ \hat{a}_{1t} = \hat{\sigma}_{1t} \varepsilon_{1t}; \quad \hat{\sigma}_{1t}^2 &= 0.005 + 0.134 \hat{a}_{1t-1}^2 + 0.856 \hat{\sigma}_{1t-1}^2 \\ (0.002) \quad (0.029) & \quad (0.030) \end{aligned}$$

where the ε_{1t} are i.i.d with a generalized error distribution (ged). This distribution was tried in order to better accommodate the existence of large (in absolute value) values (heavier tails than the normal distribution) visible in the time plots in Figure 3 (the normal and the Student-t distributions were also tried, but the ged provided better fit and better results). However, the AR(2) estimate is nonsignificant and therefore the model was refined by dropping this parameter (the GARCH(1,2), GARCH(2,1) and GARCH(2,2) models were also tried with similar results). The refined model is (standard errors in parentheses)

Equation 9

$$\begin{aligned} (1 + 0.579B)\nabla \log(\text{BTC})_t &= (1 - 0.617B)\hat{a}_{1t} \\ (0.222) & \quad (0.217) \\ \hat{a}_{1t} &= \hat{\sigma}_t \varepsilon_{1t}; \quad \hat{\sigma}_{1t}^2 = 0.005 + 0.134\hat{a}_{1t-1}^2 + 0.856\hat{\sigma}_{1t-1}^2. \\ & \quad (0.002) \quad (0.029) \quad (0.030) \end{aligned}$$

The estimates are all significant. The standardized residuals $\tilde{a}_{1t} = \hat{a}_{1t} / \hat{\sigma}_{1t}$ change randomly around a zero mean, their sample ACF and PACF only show a single significant value at a very large lag, which is irrelevant, the value of the Ljung-Box statistic for 30 lags is 29.6 with a p-value of 0.487 which is nonsignificant (and it is also nonsignificant for all the lags considered) and the Lagrange-Multiplier test statistic is 27.5 (30 lags) with a p-value of 0.594 which is nonsignificant. Furthermore, the sample ACF and PACF of the squared standardized residuals show no significant values and the Ljung-Box statistic is 28.1 with a p-value of 0.567 which is nonsignificant (and it is also nonsignificant for all the lags considered), implying that there is no ARCH effect (the plots mentioned above are not shown in order to save space). Consequently, the ARMA(1,1)-GARCH(1,1) model above is adequate for the differenced log Bitcoin price time series.

- SP500 – The sample ACF and PACF of the differenced log SP500 (Figure 4) do not show any relevant significant values, since the only significant value occurs at lag 15 which is irrelevant. The Ljung-Box statistic is 26.6 with a p-value of 0.646 for 30 lags which is nonsignificant. Therefore, this time series shows no serial correlation, i.e., $\nabla \log(\text{SP500})_t = a_{2t}$ where a_{2t} is a white noise (consequently, no model fitting was required and there are no plots in Figures 5 and 6 concerning this variable).

On the contrary, the sample ACF and PACF of the squares of the differenced log SP500 (Figure 7) show many significant values, suggesting the existence of conditional heteroscedasticity and, in fact, the plot in Figure 3 shows the volatility clusters mentioned above. Moreover, the Ljung-Box statistic for 30 lags is 267.5 with a p-value of approximately 0, i.e., highly significant (and it is significant for all the lags considered) and the Lagrange-Multiplier test statistic is 121.9 (30 lags) with

a p-value of approximately 0. Thus, the presence of conditional heteroscedasticity (ARCH effect) is confirmed. Since the sample ACF and PACF of the squared residuals both tail off, a GARCH model appeared to be appropriate. We started with a GARCH(1,1) and the fitted model is (standard errors in parentheses)

Equation 10

$$\begin{aligned} \nabla \log(\text{SP500})_t &= a_{2t} \\ \hat{a}_{2t} &= \hat{\sigma}_{2t} \varepsilon_{2t}; \quad \hat{\sigma}_{2t}^2 = 0.004 + 0.13 \hat{a}_{2t-1}^2 + 0.860 \hat{\sigma}_{2t-1}^2 \\ &\quad (0.002) \quad (0.029) \quad (0.030) \end{aligned}$$

where the ε_{2t} are i.i.d with a generalized error distribution. The estimates are all significant. The standardized residuals $\tilde{a}_{2t} = \hat{a}_{2t} / \hat{\sigma}_{2t}$ change randomly around a zero mean, their sample ACF and PACF only shows two significant values, which is irrelevant, the value of the Ljung-Box statistic for 30 lags is 32.7 with a p-value of 0.334 which is nonsignificant (and it is also nonsignificant for all the lags considered) and the Lagrange-Multiplier test statistic is 28.8 (30 lags) with a p-value of 0.53 which is also nonsignificant. Additionally, the sample ACF and PACF of the squared standardized residuals shows only a single significant value (almost nonsignificant) and the Ljung-Box statistic is 29.4 with a p-value of 0.391 which is nonsignificant (and it is also nonsignificant for all the lags considered), implying that there is no ARCH effect (the plots mentioned above are not shown in order to save space). Consequently, the GARCH(1,1) model above is adequate for the differenced log SP500.

- TIPS – The sample ACF and PACF of the differenced log TIPS suggests an MA(1) since the former cuts off after lag 1 and the latter tails off. The fitted model was (standard errors in parentheses)

Equation 11

$$\begin{aligned} \nabla \log(\text{TIPS})_t &= (1 + 0.360B) \hat{a}_{3t} \\ &\quad (0.040) \end{aligned}$$

The estimated parameter is significant and verifies the invertibility condition. The residuals displayed in Figure 5 change randomly around a zero mean and in particular do not appear to exhibit volatility clusters. The residual ACF and PACF displayed in Figure 6 shows no significant values and the Ljung-Box statistic (30 lags) is 26.7 with a p-value of 0.64, which is not significant. Therefore, the hypothesis that \hat{a}_{3t} is a white noise could not be rejected. Furthermore, the sample ACF and PACF of the squared residuals (Figure 7) shows a single significant value at a large lag which is irrelevant, the value of the Ljung-Box statistic for 30 lags is 32.5 with a p-value of 0.346 which is nonsignificant (and it is also nonsignificant for all the lags considered) and the Lagrange-Multiplier test statistic is 31.4 (30 lags) with a p-value of 0.398 which is also nonsignificant. Therefore, no evidence of conditional heteroscedasticity (ARCH effect) could be found. Consequently, the MA(1) model above is adequate for the differenced log TIPS time series.

- Unique addresses – The sample ACF and PACF of the differenced log Unique addresses suggest an MA(2) since the former cuts off after lag 2 and the latter tails off. The fitted model was (standard errors in parentheses)

Equation 12

$$\nabla \log(\text{UA})_t = (1 + 0.509B + 0.184B^2) \hat{a}_{4t}.$$

(0.042) (0.043)

The estimated parameters are significant and verify the invertibility condition. The residuals (Figure 5) show some volatility clusters already exhibited by the differenced log Unique addresses. The residual ACF and PACF (Figure 6) show only a few significant values at large lags with no relevance. These lags cause the Ljung-Box statistic to be significant with a value of 47.7 and a p-value of 0.021 (nonsignificant at a 1% level) for 30 lags. However, since these significant ACF values occur at lags with no relevance, we ignored the significance of the Ljung-Box statistic (and recall it is nonsignificant at 1% level) and did not reject the hypothesis that \hat{a}_{4t} is a white noise.

The sample ACF and PACF of the squared residuals (Figure 7) show significant values in the first lag and in a large lag, suggesting the existence of conditional heteroscedasticity. Furthermore, the Ljung-Box statistic (30 lags) is 44.1 with a p-value of 0.047 which is significant, although nonsignificant at a 1% level (and it is significant for many lags) and the Lagrange-Multiplier test statistic is 40.7 (30 lags) with a p-value of 0.091 which is nonsignificant (but significant at a 10% level). Thus, the presence of weak conditional heteroscedasticity appeared to be confirmed. Since the sample ACF and PACF of the squared residuals both exhibit a significant spike in the first lag and some high values, although nonsignificant, in the low lags, the GARCH(1,1) model was considered first. The joint estimation of an MA(2)-GARCH(1,1) model led to (standard errors in parentheses)

Equation 13

$$\begin{aligned}\nabla \log(\text{UA})_t &= (1 - 0.054B + 0.026B^2) \hat{a}_{4t} \\ &\quad (0.026) \quad (0.026) \\ \hat{a}_{4t} &= \hat{\sigma}_{4t} \varepsilon_{4t}; \quad \hat{\sigma}_{4t}^2 = 0.012 + 0.161 \hat{a}_{4t-1}^2 + 0.795 \hat{\sigma}_{4t-1}^2 \\ &\quad (0.003) \quad (0.027) \quad (0.034)\end{aligned}$$

where the ε_{4t} are i.i.d normal variables. Since only weak heteroscedasticity was detected, it appeared more appropriate to consider the normal distribution for ε_{4t} than the ged, as in the previous cases, or the Student-t. However, the estimate of the second moving average parameter is nonsignificant and therefore the model was refined by dropping it (the GARCH(1,2), GARCH(2,1) and GARCH(2,2) models were also tried but the results were worse). The refined model is (standard errors in parentheses)

Equation 14

$$\begin{aligned}\nabla \log(\text{UA})_t &= (1 - 0.055B) \hat{a}_{4t} \\ &\quad (0.266) \\ \hat{a}_{4t} &= \hat{\sigma}_{4t} \varepsilon_{4t}; \quad \hat{\sigma}_{4t}^2 = 0.011 + 0.159 \hat{a}_{4t-1}^2 + 0.798 \hat{\sigma}_{4t-1}^2 \\ &\quad (0.003) \quad (0.026) \quad (0.033)\end{aligned}$$

The estimated parameters are significant and verify the invertibility condition. The standardized residuals $\tilde{a}_{4t} = \hat{a}_{4t} / \hat{\sigma}_{4t}$ change randomly around a zero mean, their sample ACF and PACF only show a single significant value at a very large lag, which is irrelevant, the value of the Ljung-Box statistic (30 lags) is 30 with a p-value of 0.466 which is nonsignificant (and it is also nonsignificant for all the lags considered) and the Lagrange-Multiplier test statistic is 24.7 (30 lags) with a p-value of 0.738 which is nonsignificant. Furthermore, the sample ACF and PACF of the squared standardized residuals only show a single significant value at a very large lag, which is irrelevant, and the Ljung-Box statistic is 27.5 with a p-value of 0.598 which is nonsignificant (and it is also nonsignificant for all the lags considered), implying that there is no ARCH effect (the plots mentioned above are not shown). Consequently, the MA(1)-GARCH(1,1) model above is adequate for the differenced log Unique addresses time series.

- Miner fees – The sample ACF and PACF of the differenced log Miner fees suggest an MA(1) since the former cuts off after lag 1 and the latter tails off. The fitted model was (standard errors in parentheses)

Equation 15

$$\nabla \log(\text{MF})_t = (1 + 0.431B)\hat{a}_{5t}.$$

(0.051)

The estimated parameter is significant and verifies the invertibility condition. The residuals are plotted in Figure 5 and show the volatility clusters already exhibited by the differenced log Miner fees. The residual ACF and PACF displayed in Figure 6 both show only two significant values at lags with no relevance. The Ljung-Box statistic is 33.4 with a p-value of 0.304 (30 lags) which is nonsignificant. Therefore, the hypothesis that \hat{a}_{5t} is a white noise could not be rejected.

On the contrary, the sample ACF and PACF of the squared residuals (Figure 7) show some significant values, suggesting the existence of conditional heteroscedasticity. Also, the Ljung-Box statistic (30 lags) is 65.9 with a p-value of approximately 0 which is significant (and it is significant for all the lags considered)

and the Lagrange-Multiplier test statistic is 65.3 (30 lags) with a p-value of approximately 0. Therefore, the presence of conditional heteroscedasticity (ARCH effect) is confirmed. Since the sample ACF and PACF of the squared residuals both tail off, the GARCH(1,1) model was considered first. The joint estimation of an MA(1)-GARCH(1,1) led to (standard errors in parentheses)

Equation 16

$$\begin{aligned} \nabla \log(\text{MF})_t &= (1 - 0.033B)\hat{a}_{5t} \\ &\quad (0.022) \\ \hat{a}_{5t} &= \hat{\sigma}_{5t} \varepsilon_{5t}; \quad \hat{\sigma}_{5t}^2 = 0.005 + 0.133\hat{a}_{5t-1}^2 + 0.857\hat{\sigma}_{5t-1}^2 \\ &\quad (0.002) \quad (0.029) \quad (0.030) \end{aligned}$$

where the ε_{5t} are i.i.d with a generalized error distribution. However, the estimate of the moving average parameter is nonsignificant and therefore the model was refined by dropping it (the GARCH(1,2), GARCH(2,1) and GARCH(2,2) models were also tried but the results were worse). The refined model is (standard errors in parentheses)

Equation 17

$$\begin{aligned} \nabla \log(\text{MF})_t &= \hat{a}_{5t} \\ \hat{a}_{5t} &= \hat{\sigma}_{5t} \varepsilon_{5t}; \quad \hat{\sigma}_{5t}^2 = 0.004 + 0.131\hat{a}_{5t-1}^2 + 0.860\hat{\sigma}_{5t-1}^2 \\ &\quad (0.002) \quad (0.029) \quad (0.030) \end{aligned}$$

The estimates are all significant. The standardized residuals $\tilde{a}_{5t} = \hat{a}_{5t} / \hat{\sigma}_{5t}$ change randomly around a zero mean, their sample ACF and PACF show only two significant values, which is irrelevant, the value of the Ljung-Box statistic (30 lags) is 32.7 with a p-value of 0.334 which is nonsignificant (and it is also nonsignificant for all the lags considered) and the Lagrange-Multiplier test statistic is 28.8 (30 lags) with a p-value of 0.530 which is nonsignificant. In addition, the sample ACF and PACF of the squared standardized residuals show a single significant value (almost nonsignificant) and the Ljung-Box statistic is 29.4 with a p-value of 0.495 which is nonsignificant (and it is also nonsignificant for all the lags considered), implying that there is no ARCH effect (the plots mentioned above are not shown).

Consequently, the GARCH(1,1) model above is adequate for the differenced log Miner fees time series.

- Number of transactions – The sample ACF and PACF of the differenced log Number of transactions suggest an MA(2) since the former cuts off after lag 2 and the latter tails off. The fitted model was (standard errors in parentheses)

Equation 18

$$\nabla \log(\text{NTR})_t = (1 + 0.313B + 0.238B^2) \hat{a}_{6t}.$$

(0.042) (0.045)

The estimated parameters are significant and verify the invertibility condition. The residuals (Figure 5) show some volatility clusters already exhibited by the differenced log Number of transactions. The residual ACF and PACF (Figure 6) show only a few significant values at large lags with no relevance. These lags cause the Ljung-Box statistic to be significant with a value of 60.1 and a p-value of 0.001 (30 lags). However, since these significant ACF values occur at lags with no relevance, we ignored the significance of the Ljung-Box statistic and did not reject the hypothesis that \hat{a}_{6t} is a white noise.

The sample ACF and PACF of the squared residuals (Figure 7) show many significant values, suggesting the existence of conditional heteroscedasticity. The Ljung-Box statistic (30 lags) is 88.7 with a p-value of approximately 0 which is significant (and it is significant for all the lags considered) and the Lagrange-Multiplier test statistic is 71 (30 lags) with a p-value of approximately 0. Thus, the presence of conditional heteroscedasticity (ARCH effect) is confirmed. Since the sample ACF and PACF of the squared residuals both tail off, the GARCH(1,1) model was considered first. The joint estimation of an MA(2)-GARCH(1,1) led to (standard errors in parentheses)

Equation 19

$$\begin{aligned} \nabla \log(\text{NTR})_t &= (1 - 0.032B + 0.027B^2) \hat{a}_{6t} \\ &\quad (0.022) \quad (0.020) \\ \hat{a}_{6t} &= \hat{\sigma}_{6t} \varepsilon_{6t}; \quad \hat{\sigma}_{6t}^2 = 0.005 + 0.133\hat{a}_{6t-1}^2 + 0.856\hat{\sigma}_{6t-1}^2 \\ &\quad (0.002) \quad (0.029) \quad (0.030) \end{aligned}$$

where the ε_{6t} are i.i.d with a generalized error distribution. However, the estimates of the moving average parameters are not nonsignificant and therefore the model was refined by dropping them (the GARCH(1,2), GARCH(2,1) and GARCH(2,2) models were also tried with similar results). The refined model is (standard errors in parentheses)

Equation 20

$$\begin{aligned} \nabla \log(\text{NTR})_t &= \hat{a}_{6t} \\ \hat{a}_{6t} &= \hat{\sigma}_{6t} \varepsilon_{6t}; \quad \hat{\sigma}_{6t}^2 = 0.004 + 0.131\hat{a}_{6t-1}^2 + 0.860\hat{\sigma}_{6t-1}^2. \\ &\quad (0.002) \quad (0.029) \quad (0.030) \end{aligned}$$

The estimates are all significant. The standardized residuals $\tilde{a}_{6t} = \hat{a}_{6t} / \hat{\sigma}_{6t}$ change randomly around a zero mean, their sample ACF and PACF only show only two significant values (one of them is almost nonsignificant), which is irrelevant, the value of the Ljung-Box statistic (30 lags) is 32.7 with a p-value of 0.334 which is nonsignificant (and it is also nonsignificant for all the lags considered) and the Lagrange-Multiplier test statistic is 28.8 (30 lags) with a p-value of 0.53 which is nonsignificant.

What's more, the sample ACF and PACF of the squared standardized residuals show a single significant value (almost nonsignificant) and the Ljung-Box statistic is 29.4 with a p-value of 0.495 which is nonsignificant (and it is also nonsignificant for all the lags considered), implying that there is no ARCH effect (the plots mentioned above are not shown). As a result, the GARCH(1,1) model above is adequate for the differenced log Number of transactions time series.

- Gold price – The sample ACF and PACF of the differenced log gold price (Figure 4) do not show any relevant significant values, since the only two significant values occur at large lags. The Ljung-Box statistic is 35 with a p-value of 0.242 (30 lags) which is nonsignificant. Therefore, this time series shows no serial correlation, i.e. $\nabla \log(\text{GOLD})_t = a_{7t}$ where a_{7t} is a white noise (consequently, no model fitting was required and there are no plots in Figures 5 and 6 concerning this variable). Furthermore, the sample ACF and PACF of the squares of the differenced log Gold price (Figure 7) show a single significant value at a large lag which is irrelevant, the value of the Ljung-Box statistic (30 lags) is 23.4 with a p-value of 0.797 which is nonsignificant (and it is also nonsignificant for all the lags considered) and the Lagrange-Multiplier test statistic is 23.6 (30 lags) with a p-value of 0.789 which is also nonsignificant. Hence, no evidence of conditional heteroscedasticity (ARCH effect) could be found. Consequently, the differenced log Gold price time series appears to be a white noise.
- Wikipedia queries – The sample ACF of the differenced log Wikipedia queries appears to cut off after lag 1 or to tail off, whereas the PACF tails off, suggesting either an MA(1) model or an ARMA model. We tried the usual ARMA(p,q) models with $p=0,1,2$ and $q=1,2$ and the ARMA(1,1) showed the best fit (standard errors in parentheses)

Equation 21

$$\begin{array}{ccc} (1 - 0.410B) \nabla \log(\text{WIKI})_t = (1 + 0.820B) \hat{a}_{8t} & & \\ (0.110) & & (0.083) \end{array}$$

The estimated parameters are significant and verify the stationarity and invertibility conditions. The residuals (Figure 5) show some volatility clusters already exhibited by the differenced log Wikipedia queries. The residual ACF and PACF (Figure 6) show only a few significant values with no relevance and the value of the Ljung-Box statistic (30 lags) is 42.5 with a p-value of 0.065 which is nonsignificant. Therefore, the hypothesis that \hat{a}_{8t} is a white noise could not be rejected.

The sample ACF and PACF of the squared residuals (Figure 7) show several significant values, suggesting the existence of conditional heteroscedasticity. The

Ljung-Box statistic (30 lags) is 45.7 with a p-value of 0.033 which is significant (and it is highly significant for all the lags considered). The Lagrange-Multiplier test statistic is 36.1 (30 lags) with a p-value of 0.206 which is nonsignificant, but it is highly significant for other lags (for example, for 15 and for 20 lags the p-value is 0.002 and 0.013 respectively). Thus, the presence of conditional heteroscedasticity appeared to be confirmed. Since the sample ACF and PACF of the squared residuals both tail off, the GARCH(1,1) model was considered first. The joint estimation of an ARMA(1,1)-GARCH(1,1) led to (standard errors in parentheses)

Equation 22

$$\begin{aligned} (1 + 0.579B)\nabla \log(\text{WIKI})_t &= (1 - 0.617)\hat{a}_{8t} \\ (0.222) & \quad (0.217) \\ \hat{a}_{8t} &= \hat{\sigma}_{8t} \varepsilon_{8t}; \quad \hat{\sigma}_{8t}^2 = 0.005 + 0.134\hat{a}_{8t-1}^2 + 0.856\hat{\sigma}_{8t-1}^2 \\ & \quad (0.002) \quad (0.029) \quad (0.030) \end{aligned}$$

The estimated parameters are all significant and verify the stationarity and invertibility conditions. The standardized residuals $\tilde{a}_{8t} = \hat{a}_{8t} / \hat{\sigma}_{8t}$ change randomly around a zero mean, their sample ACF and PACF show no significant values, the value of the Ljung-Box statistic (30 lags) is 29.6 with a p-value of 0.487 which is nonsignificant (and it is also nonsignificant for all the lags considered) and the Lagrange-Multiplier test statistic is 27.5 (30 lags) with a p-value of 0.594 which is nonsignificant.

Additionally, the sample ACF and PACF of the squared standardized residuals show no significant values and the Ljung-Box statistic is 28.1 with a p-value of 0.567 which is nonsignificant (and it is also nonsignificant for all the lags considered), implying that there is no ARCH effect (the plots mentioned above are not shown). Consequently, the MA(1)-GARCH(1,1) model above is adequate for the differenced log Wikipedia queries time series.

Figure 7
ARMA residual plots

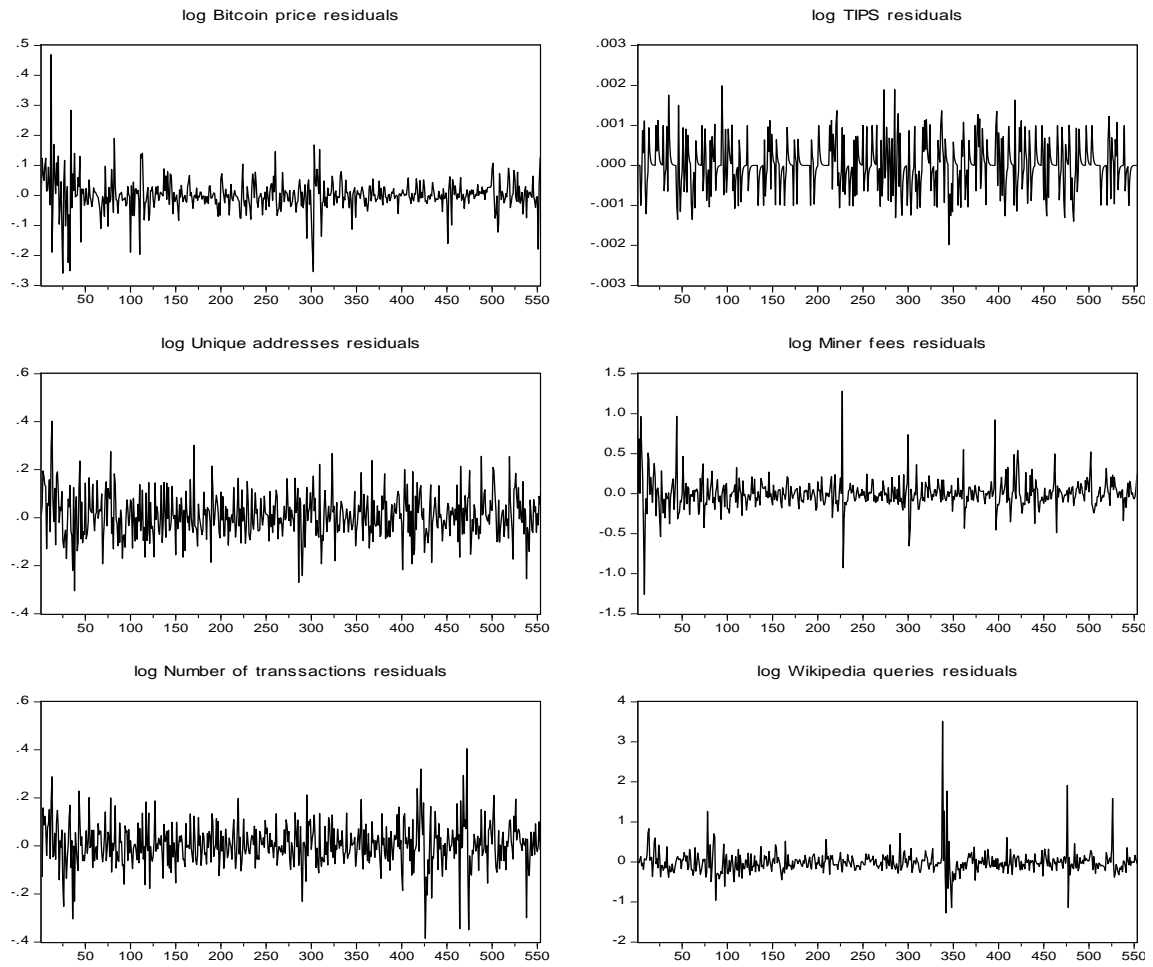
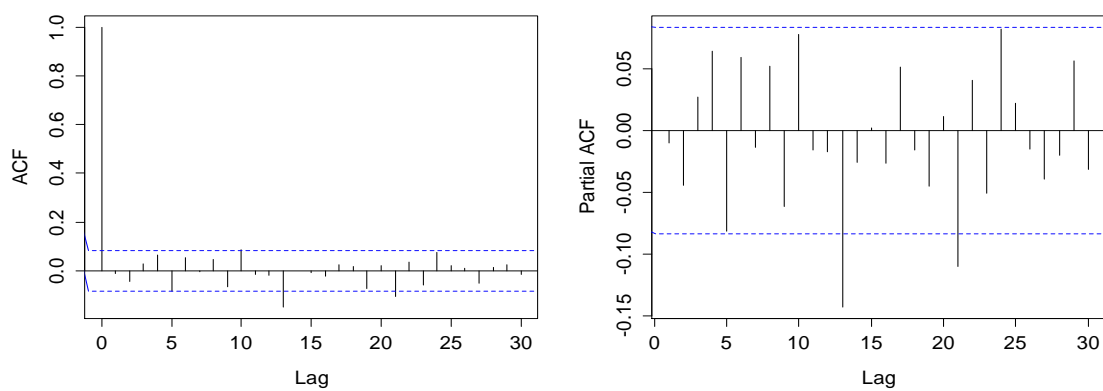
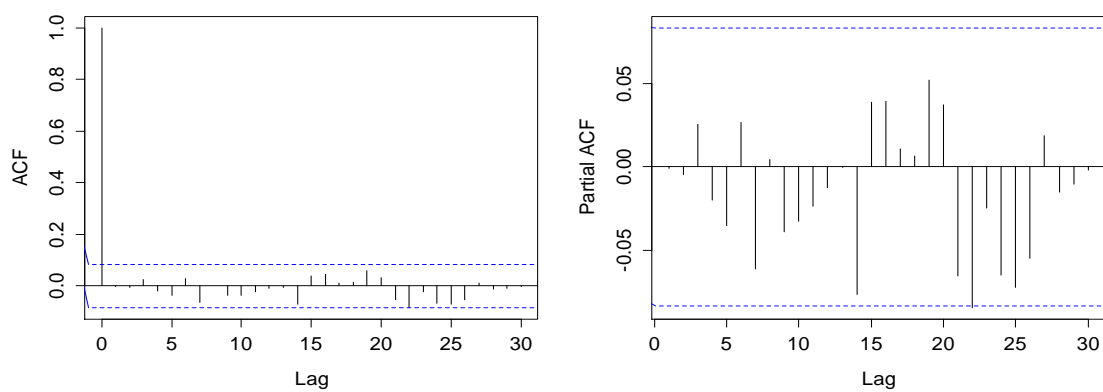


Figure 8
Residual correlograms

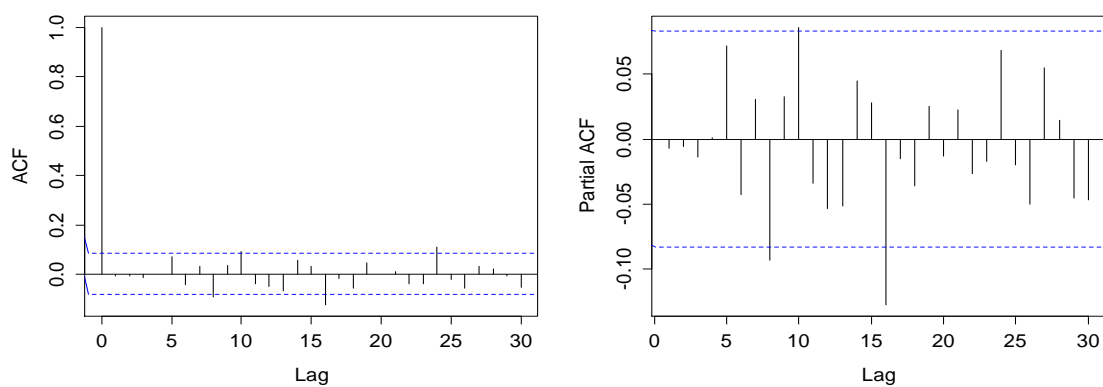
log Bitcoin price



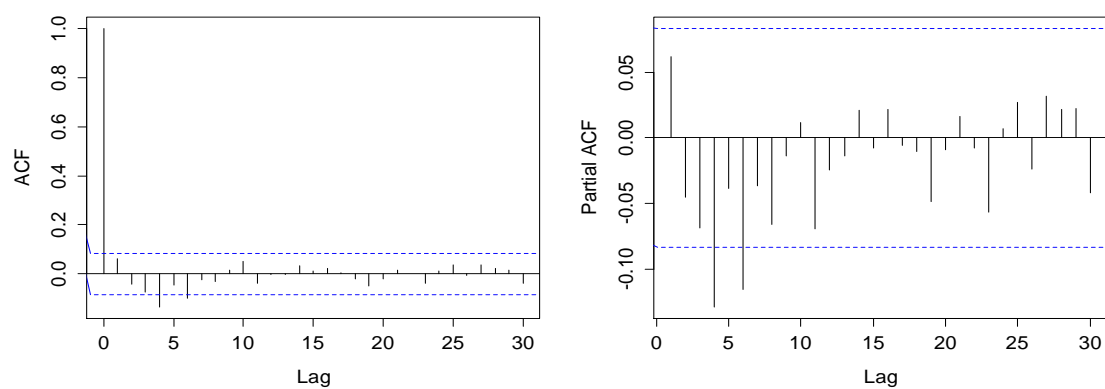
log TIPS



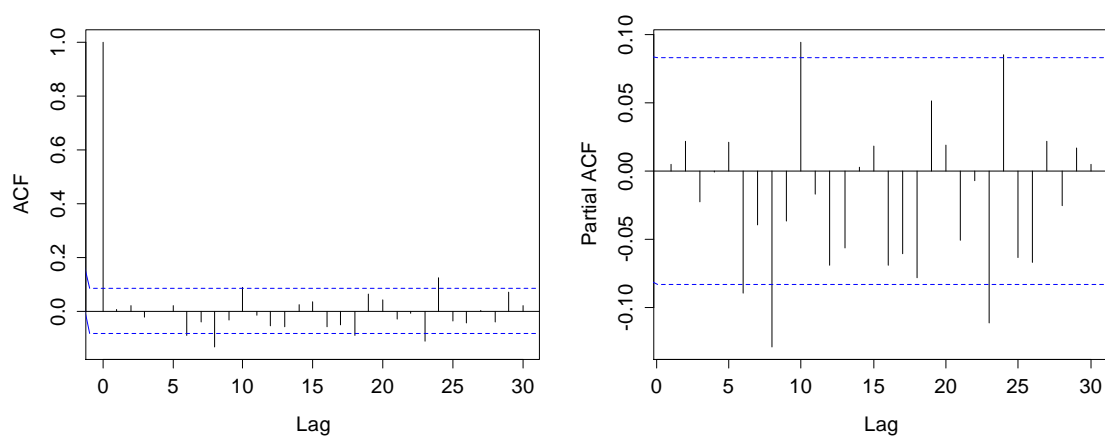
log Unique addresses



log Miner fees



log Number of transactions



log Wikipedia queries

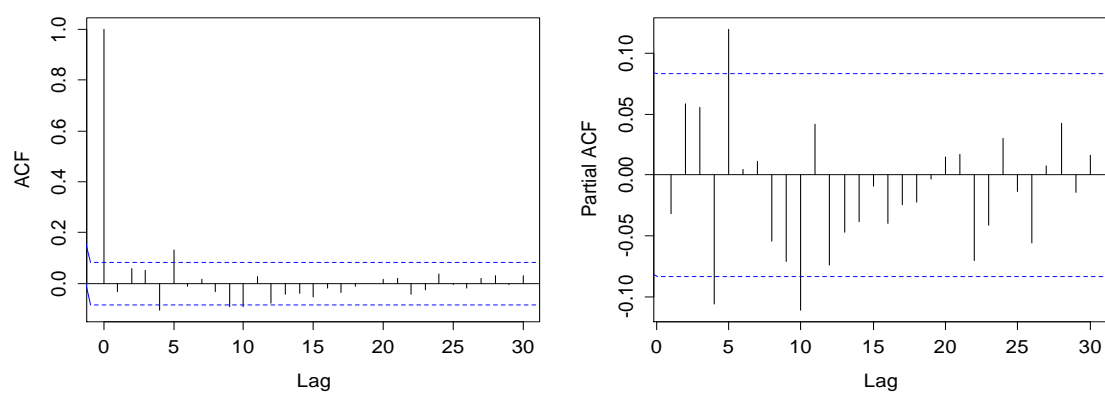
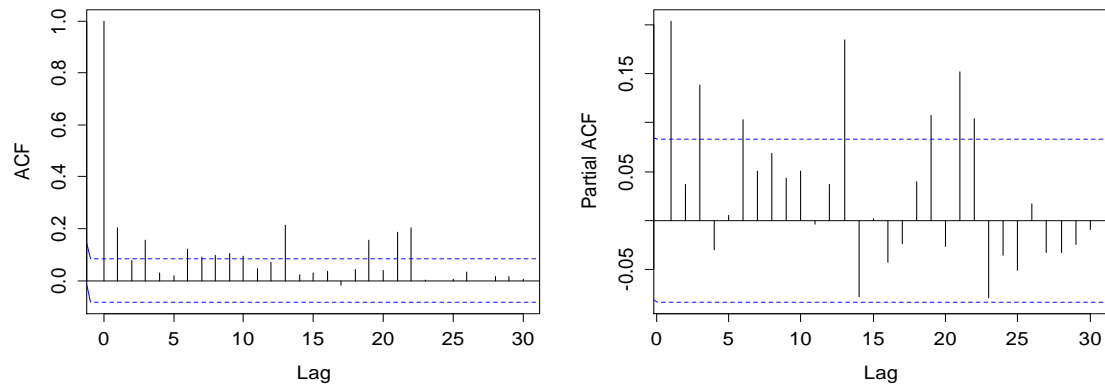
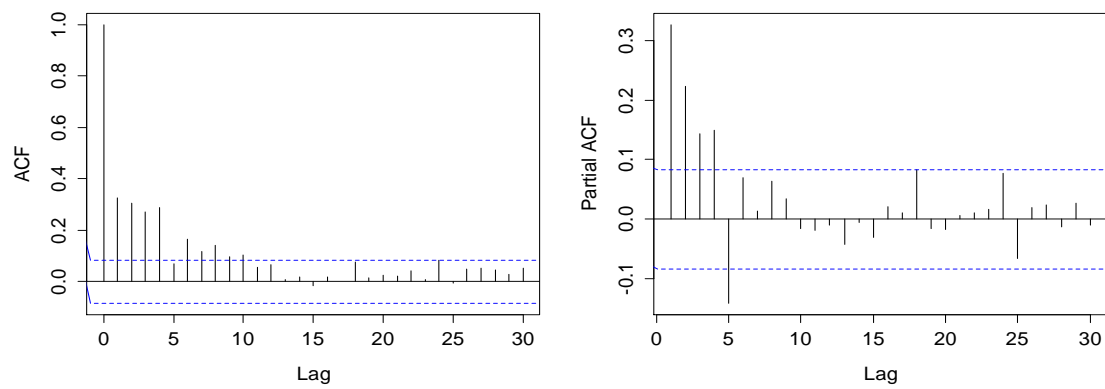


Figure 9
Correlograms of squared residuals

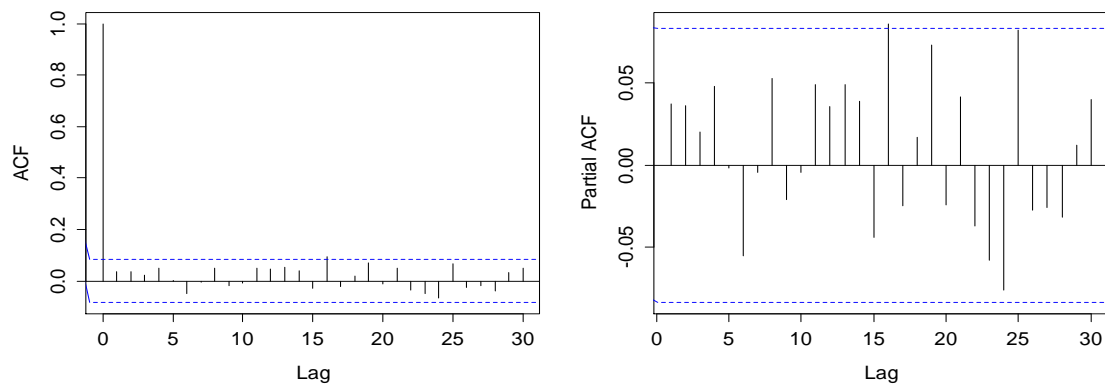
log Bitcoin price



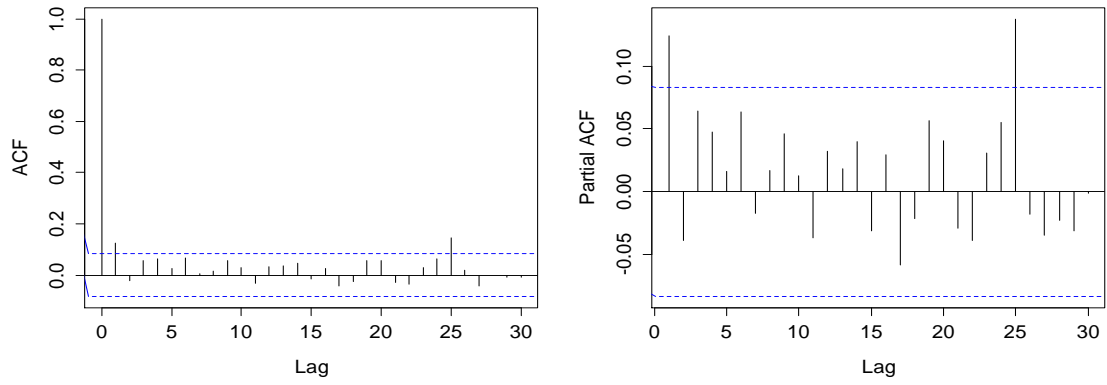
log SP500



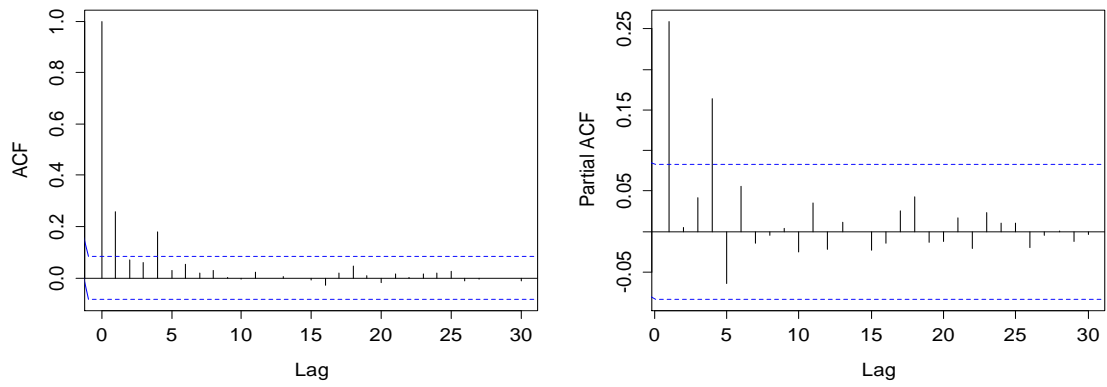
log TIPS



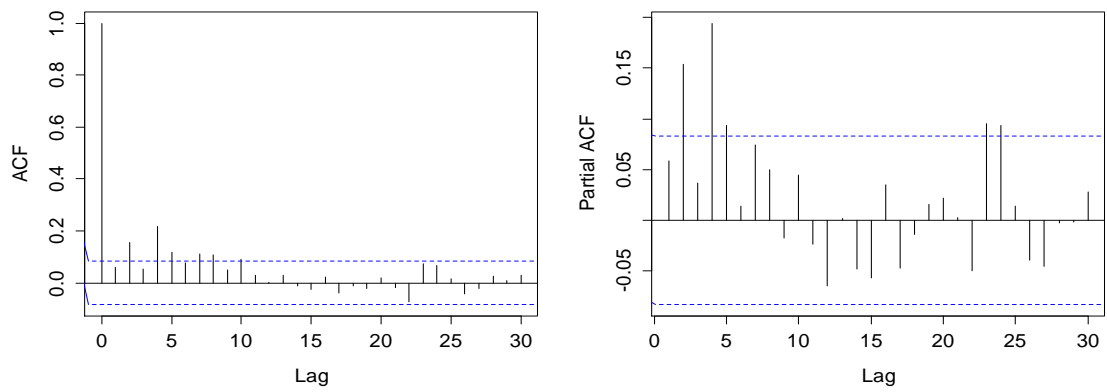
log Unique addresses



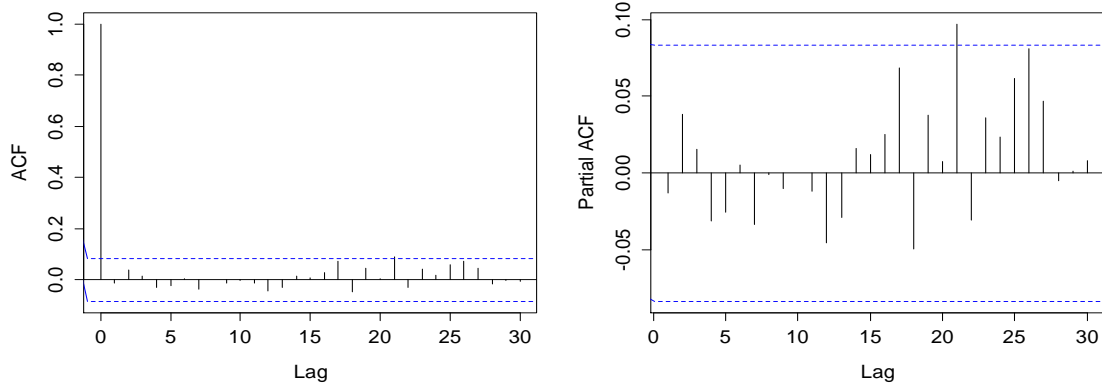
log Miner fees



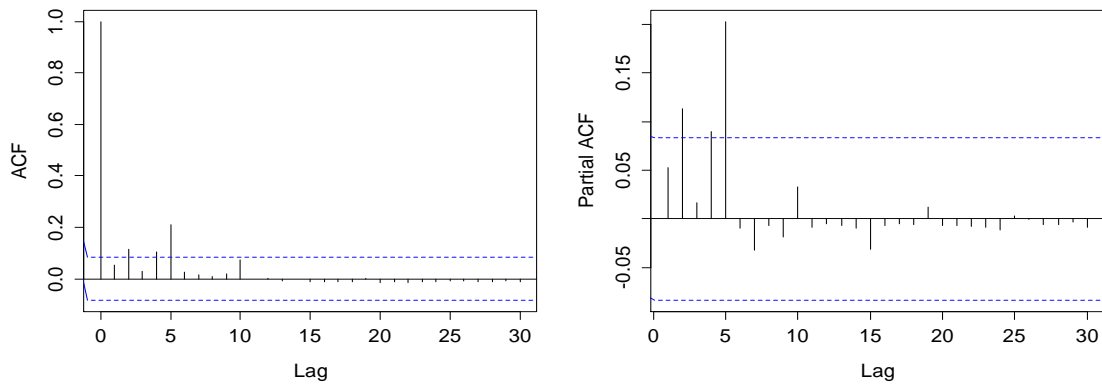
log Number of transactions



log Gold price



log Wikipedia queries



Conditional heteroscedasticity is then present in almost all the time series considered (with only two exceptions). Consequently, a regression model for the Bitcoin price with heteroscedastic errors might be appropriate.

4.2 Regression model with heteroscedastic errors

A regression model for the Bitcoin price with the other variables as regressors and heteroscedastic errors may be appropriate. Furthermore, the Bitcoin price may also depend on its volatility.

4.2.1 GARCH-in-mean model

To model such a phenomenon, we considered the GARCH-in-mean model or GARCH-M, a regression model with heteroscedastic errors and a term with the conditional

variance as an additional regressor (Engle, Lilien and Robins, 1987). Assuming a GARCH(1,1) model for the volatility equation, the fitted model with the differenced log variables is (standard errors in parentheses)

Equation 23

$$\begin{aligned} \nabla \log(\text{BTCP})_t = & -0.003 + 0.132 \nabla \log(\text{SP500})_t - 1.600 \nabla \log(\text{TIPS})_t + 0.005 \nabla \log(\text{UA})_t \\ & (0.00008) \quad (0.008) \quad (0.091) \quad (0.0004) \\ & + 0.049 \nabla \log(\text{MF})_t - 0.019 \nabla \log(\text{NTR})_t - 0.047 \nabla \log(\text{GOLD})_t - 0.001 \nabla \log(\text{WIKI})_t + 1.466 \hat{\sigma}_t^2 + \hat{a}_t \\ & (0.001) \quad (0.001) \quad (0.004) \quad (0.00008) \quad (0.043) \\ \hat{a}_t = \hat{\sigma}_t \varepsilon_t; \quad \hat{\sigma}_t^2 = & 0.0002 + 0.224 \hat{a}_{t-1}^2 + 0.722 \hat{\sigma}_{t-1}^2 \\ & (0.00001) (0.009) \quad (0.012) \end{aligned}$$

where the ε_t are i.i.d with a generalized error distribution. The estimates are all significant. The standardized residuals $\tilde{a}_t = \hat{a}_t / \hat{\sigma}_t$ plotted in Figure 7 change randomly around a zero mean (with only a few extreme values of no relevance), their sample ACF and PACF (Figure 8) only show two significant values (the second occurs at a large lag), which is irrelevant, the value of the Ljung-Box statistic (30 lags) is 32.4 with a p-value of 0.35 which is nonsignificant and the Lagrange-Multiplier test statistic is 26.5 (30 lags) with a p-value of 0.652 which is nonsignificant. Also, the sample ACF and PACF of the squared standardized residuals (Figure 9) show a single significant value (almost nonsignificant) and the Ljung-Box statistic is 12.1 with a p-value of 0.998 which is nonsignificant, implying that there is no ARCH effect. Consequently, the GARCH-M model above is adequate.

Figure 10
Standardized residual plot

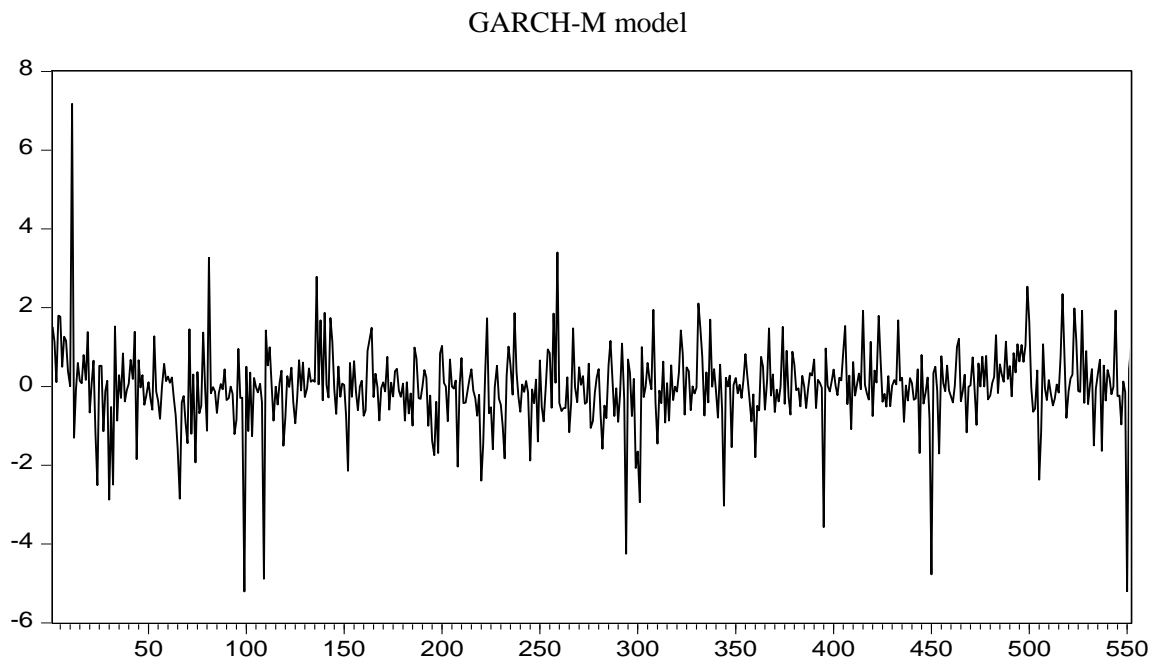


Figure 11
Residual correlograms

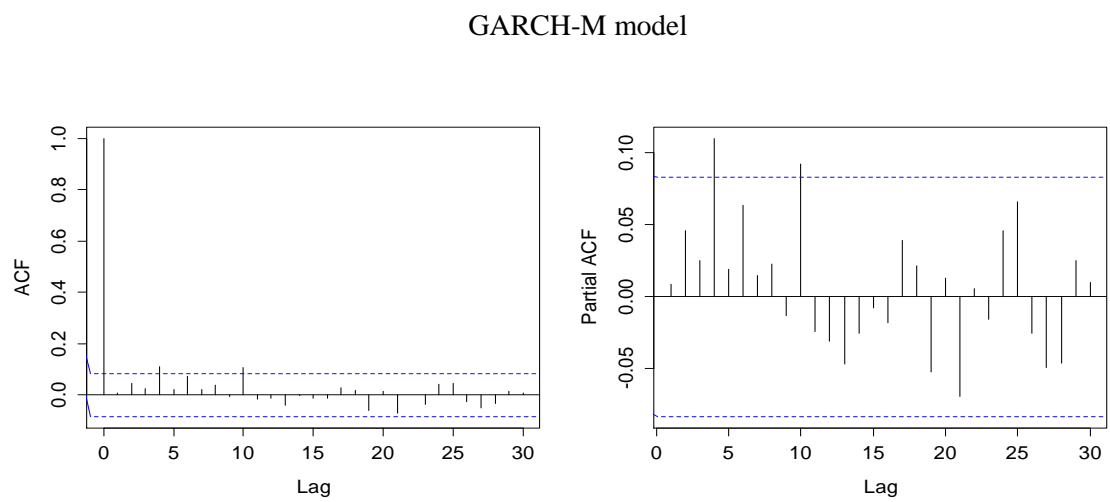
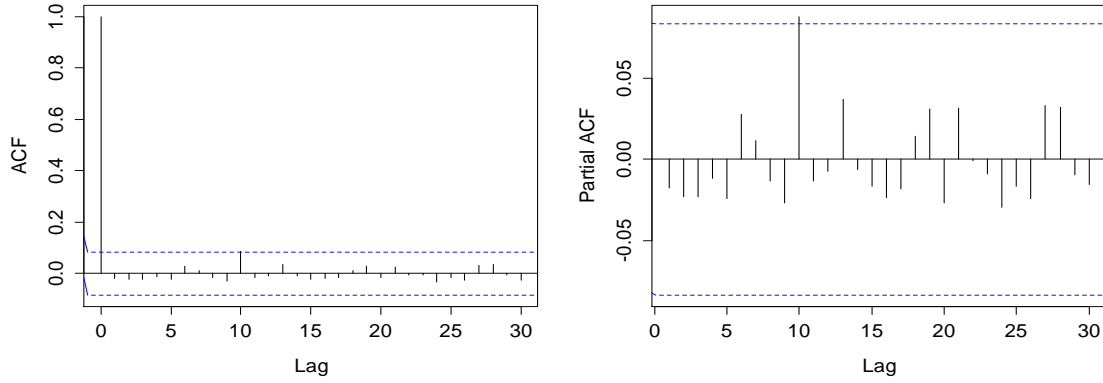


Figure 12
Correlograms of the squared residuals

GARCH-M model



Since the model is defined in terms of the differenced log variables, the interpretation of the values of the parameter estimates is not very meaningful and consequently the main interest lies on their signs. The fitted model shows that the Bitcoin price changes depend positively on the changes of the SP500, of the Unique addresses and of the Miner fees and depend negatively on the changes of the TIPS, of the Number of transactions, of the gold price and of the Wikipedia queries.

We note that the results obtained in this step generally agree with those in the cointegration regression obtained above in the VEC model. There are two main differences, nevertheless. First, the estimated coefficient of the SP500 is now significant (it was not in the cointegration equation) and therefore this variable is included in the model. The other difference is the negative sign of the estimated coefficient of the Wikipedia queries which is positive in the cointegration equation.

Moreover, another very important feature of the regression model above is the estimated parameter of the conditional variance. This estimate is highly significant and positive (correct sign) showing that there is in fact a positive effect of volatility on the Bitcoin price, i.e., there is a risk premium on the price. In other words, the Bitcoin price is positively related to its volatility.

4.2.2 Exponential GARCH-in-mean model

The effect of errors on the conditional variance is symmetric for GARCH models, i.e., a positive error has the same effect as a negative error of the same magnitude which is a weakness of these models when handling financial time series. To accommodate the asymmetric relation between many financial variables and their volatility changes, Nelson (1991) proposed the Exponential GARCH model or EGARCH.

Accordingly, we also tried an EGARCH(1,1) model for the conditional variance and the fitted model is (standard errors in parentheses)

Equation 24

$$\begin{aligned} \nabla \log(\text{BTCP})_t = & -0.004 + 0.067 \nabla \log(\text{SP500})_t - 1.804 \nabla \log(\text{TIPS})_t + 0.013 \nabla \log(\text{UA})_t \\ & (0.0002) (0.020) \quad (0.298) \quad (0.0008) \\ & + 0.051 \nabla \log(\text{MF})_t - 0.018 \nabla \log(\text{NTR})_t - 0.091 \nabla \log(\text{GOLD})_t + 0.0003 \nabla \log(\text{WIKI})_t \\ & (0.001) \quad (0.001) \quad (0.014) \quad (0.00006) \\ & + 1.928 \hat{\sigma}_t^2 + \hat{a}_t \\ & (0.098) \\ \hat{a}_t = \hat{\sigma}_t \varepsilon_t; \quad \log(\hat{\sigma}_t^2) = & -0.373 + 0.393 \frac{|\hat{a}_{t-1}|}{\hat{\sigma}_{t-1}} - 0.056 \frac{\hat{a}_{t-1}}{\hat{\sigma}_{t-1}} + 0.940 \log(\hat{\sigma}_{t-1}^2) \\ & (0.011) (0.027) \quad (0.010) \quad (0.003) \end{aligned}$$

where the ε_t are i.i.d with a generalized error distribution. The estimates are all significant. The standardized residuals $\tilde{a}_t = \hat{a}_t / \hat{\sigma}_t$ (Figure 10) change randomly around a zero mean (with only a few extreme values of no relevance), their sample ACF only shows two significant values and the PACF shows a single one, which is irrelevant; the value of the Ljung-Box statistic (30 lags) is 35.5 with a p-value of 0.226 which is nonsignificant and the Lagrange-Multiplier test statistic is 18.6 (30 lags) with a p-value of 0.947 which is nonsignificant. In addition, the sample ACF and PACF of the squared standardized residuals do not show any significant values and the Ljung-Box statistic is 8 with a p-value of approximately 1 which is nonsignificant, implying that there is no ARCH effect. Consequently, the EGARCH-M model above is adequate and provides a small improvement over the GARCH-M model.

Figure 13
Standardized residual plot

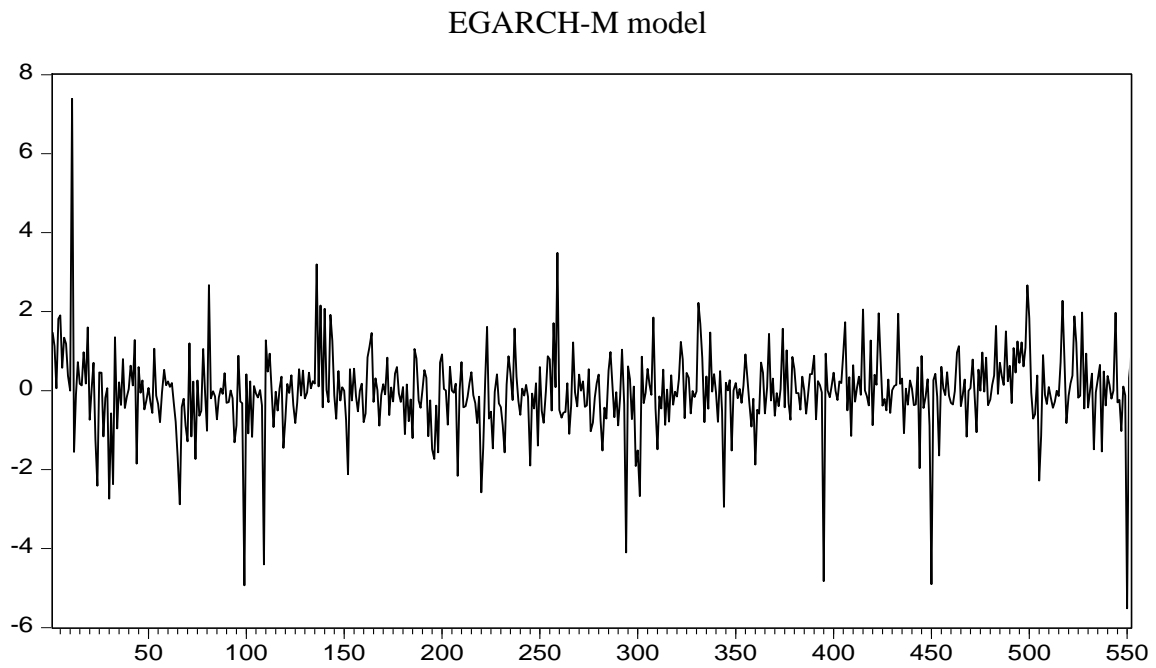


Figure 14
Residual correlograms

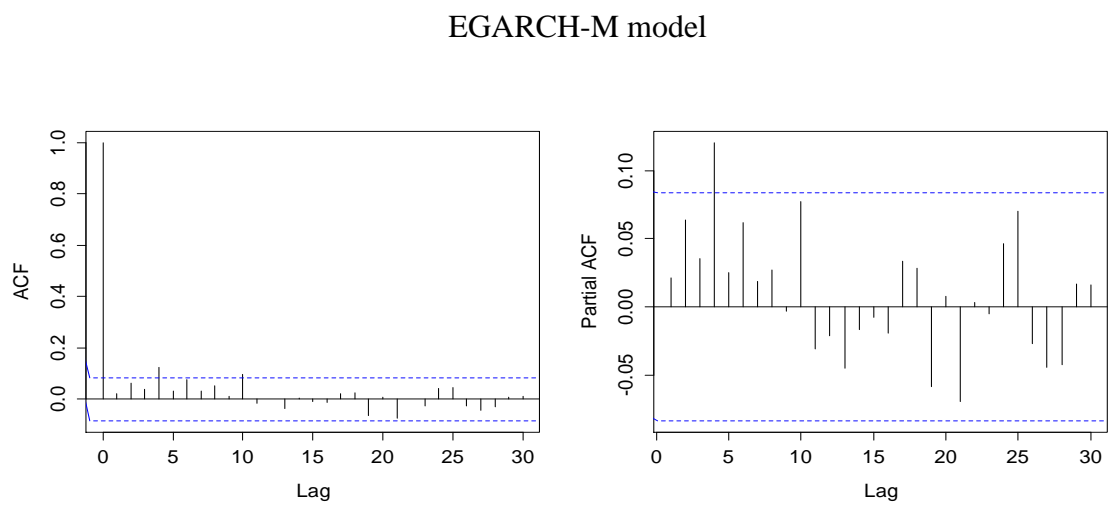
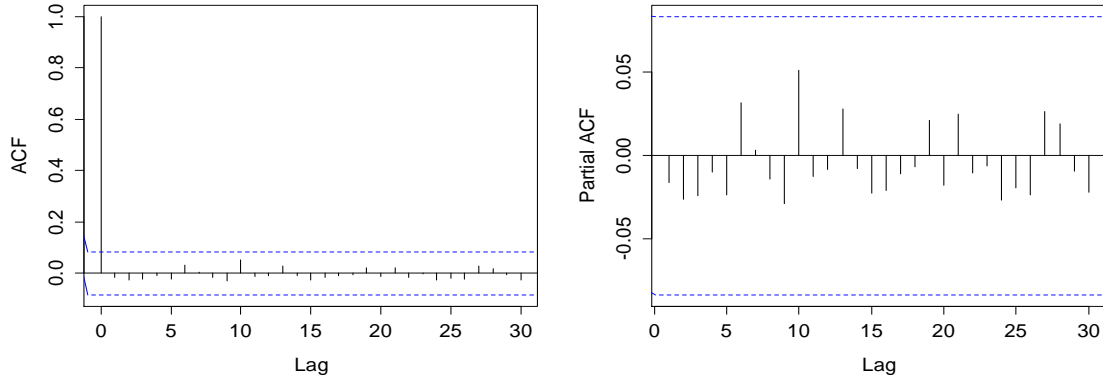


Figure 15
Correlograms of the squared residuals

EGARCH-M model



The regression parameter estimates have similar values and generally keep the same sign as in the GARCH-M model with the exception of the Wikipedia queries whose estimated coefficient is now positive, agreeing with the result obtained in the cointegration equation above. Note also that the estimated parameter of the standardized residual in the variance equation is significant and negative (-0.056) as expected, showing the asymmetric feature of EGARCH models, i.e., a negative shock has a stronger impact on volatility than a positive one. Since this model fits the data better than the GARCH-M, it appeared that its conclusions are also more accurate.

4.2.3 Threshold GARCH-in-mean model

Another volatility model commonly used to handle the asymmetric relation between many financial variables and their volatility changes is the Threshold GARCH or TGARCH proposed by Glosten, Jagannathan and Runkle (1993) and also known as GJR-GARCH (named after its authors).

Therefore, we also tried a TGARCH(1,1) model for the conditional variance and the fitted model is (standard errors in parentheses):

Equation 25

$$\begin{aligned}
\nabla \log(\text{BTCP})_t &= -0.003 + 0.151 \nabla \log(\text{SP500})_t - 1.654 \nabla \log(\text{TIPS})_t + 0.004 \nabla \log(\text{UA})_t \\
&\quad (0.0001) \quad (0.016) \quad (0.279) \quad (0.0005) \\
&+ 0.050 \nabla \log(\text{MF})_t - 0.024 \nabla \log(\text{NTR})_t - 0.051 \nabla \log(\text{GOLD})_t - 0.001 \nabla \log(\text{WIKI})_t + 1.257 \hat{\sigma}_t^2 + \hat{a}_t \\
&\quad (0.001) \quad (0.001) \quad (0.008) \quad (0.0002) \quad (0.057) \\
\hat{a}_t &= \hat{\sigma}_t \varepsilon_t; \quad \hat{\sigma}_t^2 = 0.005 + [0.259 + 0.127 I(\hat{a}_{t-1} < 0)] \hat{a}_{t-1}^2 + 0.735 \log \hat{\sigma}_{t-1}^2 \\
&\quad (0.0003) \quad (0.008) \quad (0.019) \quad (0.010)
\end{aligned}$$

where the ε_t are i.i.d with a generalized error distribution and $I(\hat{a}_{t-1} < 0)$ is the indicator function for negative \hat{a}_{t-1} , i.e.,

Equation 26

$$I(\hat{a}_{t-1} < 0) = \begin{cases} 1 & \text{if } \hat{a}_{t-1} < 0 \\ 0 & \text{if } \hat{a}_{t-1} \geq 0. \end{cases}$$

The estimates are all significant. The standardized residuals $\tilde{a}_t = \hat{a}_t / \hat{\sigma}_t$ (Figure 13) change randomly around a zero mean (with only a few extreme values of no relevance), their sample ACF and PACF (Figure 14) only show two significant values, which is irrelevant, the value of the Ljung-Box statistic (30 lags) is 36.8 with a p-value of 0.183 which is nonsignificant and the Lagrange-Multiplier test statistic is 26.9 (30 lags) with a p-value of 0.628 which is nonsignificant. Furthermore, the sample ACF and PACF of the squared standardized residuals (Figure 15) show a single significant value and the Ljung-Box statistic is 13.9 with a p-value of 0.995 which is nonsignificant, implying that there is no ARCH effect. Consequently, the TGARCH-M model above is adequate. Its fit, however, is slightly worse than with the two previous models.

Figure 16
Standardized residual plot

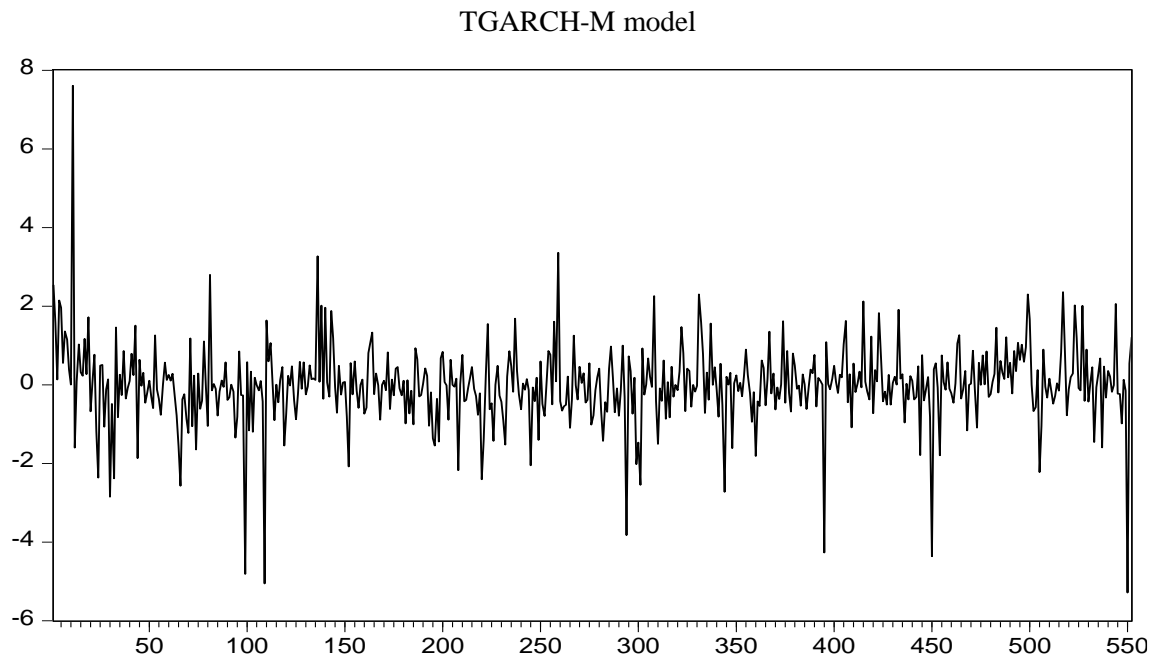


Figure 17
Residual correlograms

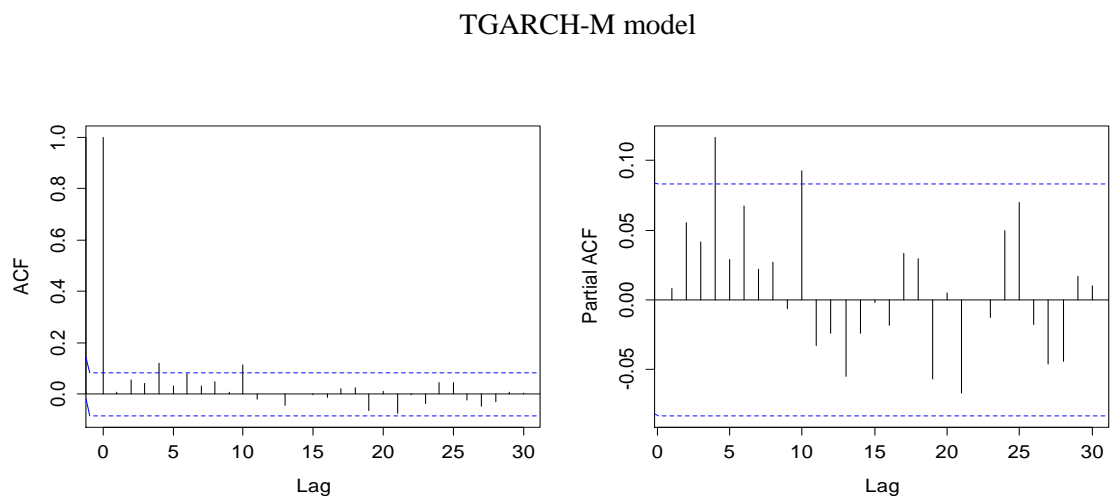
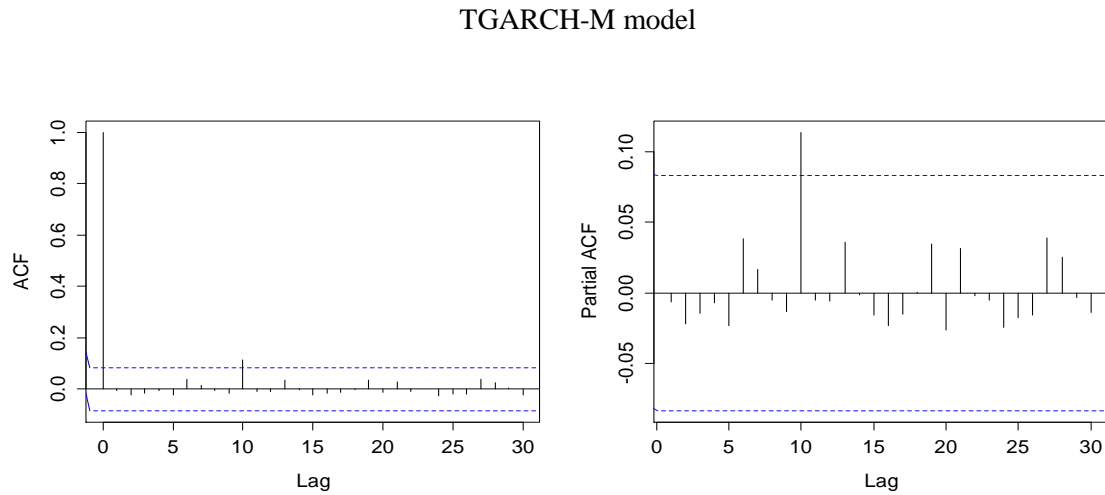


Figure 18
Correlograms of the squared residuals



The regression parameter estimates are not generally very far from those obtained with the EGARCH model, although some values show important differences. The only exception is again the Wikipedia queries whose estimated coefficient is negative one more time, as in the GARCH-M model and contradicting the result obtained with the EGARCH-M. The estimated parameter of the indicator function in the variance equation is significant and positive (0.127) as expected, showing again that a negative shock has a stronger impact on volatility than a positive one, reflecting the asymmetric feature of TGARCH models.

Comparing the three GARCH-M models, the EGARCH-M provides the best fit with more accurate conclusions.

Conclusions

Bitcoin has certainly sparked the interest of investors, crypto-currency enthusiasts and the more common public, whether it is seen as a risky investment or as a new and innovative type of currency. As a wider acceptance grows around the Bitcoin, its price dynamics has been the subject of increasing attempts at analyzing its formation. In this dissertation we study the relationship between the price and several variables, some specifically related to the Bitcoin structure and others related to market forces, giving our contribute to the growing literature around this topic.

First, we establish a relationship between the behavior of the variables and their impact on the Bitcoin price, as well as their relevance for it, given the VEC model results. Of all the variables presented, only the index for Standard's and Poor 500 was found to be irrelevant. The real yields on Treasury Inflation Protected Securities show an opposite relation with the Bitcoin price, supported by the notion that increases in the real yield provide increasing returns on financial assets over inflation, justifying bigger demand over other assets. TIPS show the strongest effect among the remaining variables used. A positive relation between unique addresses and price was found, as expected from the classical economic theory of demand and supply, given that unique addresses may be considered as a proxy for the global demand for Bitcoin; likewise, miner fees also exhibit a positive relation with the price. Given that the rewards of Bitcoin mining are measured in Bitcoins themselves, this correlation is to be expected. Surprisingly, both the number of transactions and the daily price of gold have a negative relationship with the price. We believe that increases in the number of transactions are more likely to occur in times of less volatility on price changes, therefore showing opposite directions. A substitution effect between gold and Bitcoin, considering both as "refuge assets", would partly explain this effect. Lastly, and in accordance to literature already discussed in previous chapters, increases in Wikipedia queries were found to be positively related to increases in Bitcoin price.

Second, we find that deviations above a long-run equilibrium for the Bitcoin price cause price decreases, i.e., a return to the equilibrium, a decrease in miner fees and also a decrease in the number of transactions. Symmetric effects also occur concerning deviations below the equilibrium.

Last, and considering that volatility can also have an effect on price formation, we show that, not only is there evidence of a risk premium, i.e, there is a positive effect of volatility on the Bitcoin price, but there is also confirmation that negative shocks have a stronger impact on volatility than positive ones. These conclusions, provided by generalized autoregressive conditional heteroscedastic models (E-GARCH), support the results concerning the effects of the different variables on the Bitcoin price evolution which is very encouraging.

We believe that future research may build up on the fundamental drivers of Bitcoin, either by adding new market indicators or by combining new dimensions to the price equation. As new data is collected on a daily basis, upcoming research can consider the influence of the use of other crypto currencies or even the influence of global scale events, with or without a financial nature.

Appendix

Estimated short-term autoregressive parameter matrices in the VEC(3) model (standard errors in parentheses):

Equation 27

$$\hat{\Phi}_1 = \begin{pmatrix} -0.096 & 0.025 & -1.875 & 0.026 & -0.051 & 0.002 & -0.022 & 0.006 \\ (0.045) & (0.295) & (5.122) & (0.045) & (0.017) & (0.048) & (0.293) & (0.008) \\ 0.013 & 0.031 & -0.597 & 0.001 & -0.002 & 0.004 & -0.026 & -0.003 \\ (0.007) & (0.045) & (0.782) & (0.007) & (0.003) & (0.007) & (0.045) & (0.001) \\ 0.0003 & -0.003 & -0.164 & 0.000 & 0.000 & 0.000 & -0.002 & 0.000 \\ (0.0004) & (0.003) & (0.045) & (0.000) & (0.000) & (0.000) & (0.003) & (0.000) \\ 0.174 & -0.810 & -10.638 & -0.346 & 0.023 & -0.133 & 0.572 & 0.023 \\ (0.073) & (0.477) & (8.281) & (0.072) & (0.027) & (0.078) & (0.473) & (0.013) \\ 0.769 & -0.629 & -31.706 & 0.017 & -0.353 & 0.147 & 0.097 & 0.016 \\ (0.140) & (0.916) & (15.893) & (0.139) & (0.052) & (0.150) & (0.909) & (0.026) \\ 0.262 & -0.212 & -8.848 & -0.137 & -0.013 & -0.147 & 0.534 & 0.008 \\ (0.070) & (0.454) & (7.876) & (0.069) & (0.026) & (0.075) & (0.450) & (0.013) \\ -0.002 & 0.037 & -2.607 & -0.019 & 0.000 & 0.012 & -0.019 & 0.000 \\ (0.007) & (0.045) & (0.773) & (0.007) & (0.003) & (0.007) & (0.044) & (0.001) \\ 0.109 & -4.846 & 32.081 & 0.887 & 0.124 & -0.600 & -3.196 & -0.394 \\ (0.242) & (1.576) & (27.345) & (0.239) & (0.089) & (0.259) & (1.563) & (0.044) \end{pmatrix};$$

Equation 28

$$\hat{\Phi}_2 = \begin{pmatrix} 0.069 & 0.023 & -5.574 & 0.034 & -0.044 & 0.005 & 0.147 & 0.010 \\ (0.047) & (0.297) & (5.150) & (0.041) & (0.017) & (0.044) & (0.291) & (0.009) \\ -0.011 & -0.039 & 0.261 & 0.000 & 0.002 & 0.005 & -0.085 & -0.001 \\ (0.007) & (0.045) & (0.786) & (0.006) & (0.003) & (0.007) & (0.044) & (0.001) \\ 0.000 & -0.003 & -0.059 & 0.000 & 0.000 & -0.001 & -0.001 & 0.000 \\ (0.000) & (0.003) & (0.045) & (0.000) & (0.000) & (0.000) & (0.003) & (0.000) \\ -0.014 & -0.718 & 1.797 & -0.282 & 0.023 & -0.071 & 0.119 & -0.012 \\ (0.076) & (0.480) & (8.326) & (0.066) & (0.028) & (0.072) & (0.471) & (0.014) \\ 0.220 & 0.277 & -25.444 & -0.011 & -0.168 & 0.009 & 0.148 & 0.002 \\ (0.146) & (0.921) & (15.980) & (0.128) & (0.054) & (0.138) & (0.904) & (0.027) \\ -0.044 & -0.676 & 0.815 & -0.103 & -0.007 & -0.155 & -0.189 & -0.011 \\ (0.073) & (0.456) & (7.919) & (0.063) & (0.027) & (0.068) & (0.448) & (0.013) \\ -0.013 & 0.055 & -1.150 & -0.005 & -0.002 & -0.001 & -0.048 & -0.002 \\ (0.007) & (0.045) & (0.777) & (0.006) & (0.003) & (0.007) & (0.044) & (0.001) \\ 0.106 & 1.817 & 53.007 & 0.770 & 0.066 & -0.870 & 0.081 & -0.205 \\ (0.252) & (1.584) & (27.494) & (0.219) & (0.092) & (0.237) & (1.555) & (0.046) \end{pmatrix};$$

Equation 67

$$\hat{\Phi}_3 = \begin{pmatrix} 0,055 & 0,135 & -2,621 & -0,016 & 0,002 & -0,001 & -0,252 & 0,014 \\ (0,048) & (0,299) & (5,073) & (0,036) & (0,016) & (0,040) & (0,288) & (0,008) \\ -0,001 & -0,018 & 0,614 & 0,002 & -0,001 & 0,002 & -0,029 & -0,001 \\ (0,007) & (0,046) & (0,774) & (0,005) & (0,002) & (0,006) & (0,044) & (0,001) \\ 0,000 & 0,003 & 0,009 & 0,000 & 0,000 & 0,000 & 0,000 & 0,000 \\ (0,000) & (0,003) & (0,044) & (0,000) & (0,000) & (0,000) & (0,003) & (0,000) \\ 0,057 & 0,077 & -4,527 & -0,209 & 0,009 & 0,050 & -0,316 & -0,023 \\ (0,077) & (0,483) & (8,202) & (0,058) & (0,025) & (0,065) & (0,466) & (0,013) \\ 0,378 & 1,433 & -32,067 & -0,061 & -0,023 & -0,040 & 0,457 & 0,019 \\ (0,148) & (0,926) & (15,741) & (0,111) & (0,049) & (0,124) & (0,895) & (0,025) \\ 0,047 & 0,415 & -14,451 & -0,133 & 0,022 & -0,038 & -0,605 & -0,013 \\ (0,073) & (0,459) & (7,801) & (0,055) & (0,024) & (0,062) & (0,443) & (0,012) \\ -0,005 & 0,076 & -1,844 & -0,005 & 0,000 & -0,001 & 0,006 & -0,002 \\ (0,007) & (0,045) & (0,766) & (0,005) & (0,002) & (0,006) & (0,044) & (0,001) \\ -0,159 & 0,946 & -7,250 & 0,643 & 0,062 & -0,605 & -1,504 & -0,057 \\ (0,254) & (1,594) & (27,085) & (0,190) & (0,084) & (0,214) & (1,539) & (0,043) \end{pmatrix}.$$

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